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Optimal concession contract between a port authority and container-terminal operators by revenue-sharing schemes with quantity discount

Yanjie Zhou and Kap Hwan Kim

Department of Industrial Engineering, Pusan National University, Busan, Korea

ABSTRACT

This paper proposes a method for designing an optimal concession contract under various revenue-sharing schemes with a quantity discount between a port authority and two container-terminal operators. The revenue-sharing scheme with an incremental or all-unit quantity discount provides a discount on the unit fee per container when the amount of cargo of a container terminal is over a predefined breakpoint, which is one of the popular methods for boosting the traffic volume of a port. This study defines a Stackelberg two-stage game model, in which the port authority determines the parameters of the revenue-sharing scheme to maximize its total revenue in the first stage, and two container-terminal operators compete with each other to maximize their profit by determining the terminal handling charge in the second stage. Numerical experiments show that the revenue-sharing scheme with a quantity discount results in higher revenue to the port authority than that from the traditional revenue-sharing scheme with a single rate. Moreover, revenue sharing with an all-unit discount provides higher revenue than that with an incremental discount in almost all the experimental results.

KEYWORDS

Concession contract; Stackelberg game; revenue sharing; port authority; container-terminal operators; quantity discount

1. Introduction

According to the United Nations Conference on Trade and Development (UNCTAD 2018), the total volume of seaborne trade reached 10.7 billion tons and the global containerized trade increased by 6.4% in 2018. UNCTAD forecasted that the annual growth rate of volume would be 3.8% between 2018 and 2023 due to the continued growth of the global economy. In response to the increasing demand of container cargo, the capacities of container terminals are being expanded and new container terminals are being constructed in many countries. On the other hand, expanding the capacities of ports or constructing new ports breaks the original balance of container volume distribution and in some regions, results in severe competition among neighboring container terminals. For example, the terminal handling charge (THC) in Busan, Korea, dropped by 21.5% on average after the Busan New Port began operation in 2010.

A landlord port authority, which is a popular type of port development, lends container terminals to container-terminal operators, who pay a rental fee to the port authority according to their concession contracts, set their terminal handling charges, and provide cargo-handling services

CONTACT Kap Hwan Kim kapkim@pusan.ac.kr Department of Industrial Engineering, Pusan National University, 30 Jangjeon-dong, Gumjeong-gu, Busan, South Korea

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to their customers. Setting the concession contract is important for the landlord port authorities because it affects the management strategies of the container-terminal operators as well as the revenue of the port authorities. Because the port authorities are financially self-supporting, the most popular short-term objective of a landlord port authority is to maximize the total revenue collected from container-terminal operators.

Concession contracts can be classified into a lump-sum, annual rent, revenue sharing, and mixed policies. The lump-sum contract is a traditional type of contract, in which a fixed amount of money is paid in one installment or a series of payments over time. Korean port authorities have used lump-sum type contracts for infrastructure, dredging, breakwater, and water channels (Lee and Lee 2012). In the annual rent scheme, a container-terminal operator who leases a container terminal from the port authority pays a fixed rental fee every year. This scheme is also one of the many popular schemes. Chen and Liu (2014) called the annual rent scheme the fixed fee scheme.

In a revenue-sharing scheme, the total amount of income obtained by a container-terminal operator is shared between the container-terminal operator and the port authority. Revenue sharing, which is a very popular policy in the supply chain (Feng, Moon, and Ryu 2014), was first adopted by the port authorities in the Indian subcontinent and has also been used in some small ports in the Philippines (Farrell 2011). According to a survey by Notteboom (2008), who examined various concession fee schemes for 43 container-terminal operators in European seaports, revenue-sharing schemes reached 25% of all schemes used for container-terminal operators.

The revenue-sharing scheme is advantageous to container terminals in that the amount of payment is determined based on the capability of the payment, while the port authority has an incentive to promote the port to the market. The most popular way of revenue sharing is to collect a fixed unit fee per twenty-foot-equivalent-units (TEUs) handled by a container-terminal operator, which is called the unit fee scheme (Chen and Liu 2014). The Karachi Port in Pakistan (Saeed and Larsen 2010b) and the Chittagong Port in Bangladesh (Munim, Saeed and Larsen 2019) adopted a revenue-sharing policy with a unit fee scheme. Farrell (2011) examined 85 container terminals with open-source data on a concession fee and found that 60% of the container terminals use one of the above schemes (pure policy) but 40% of the container terminals use a combination of different schemes (mixed policy).

In many revenue-sharing schemes by some port authorities, multiple rates per TEU are being used instead of a single fixed rate (unit fee) per TEU. This revenue sharing is a generalized version of the unit fee scheme and is called revenue sharing with a quantity discount, which is the main issue of this paper. Revenue sharing with a quantity discount is a popular scheme used in European seaports. If the total amount of handled cargo exceeds a predefined threshold, the container-terminal operator receives a discount for the unit fee in the rental fee. Lee and Lee (2012) reported a real example of revenue sharing with a quantity discount in Korea.

This study first investigated how to design the parameters of the concession contract using the revenue-sharing schemes by a quantity discount. This paper proposes a model of a Stackelberg two-stage game. In the first stage, the port authority first sets the concession parameters to maximize its own total revenue collected from container-terminal operators. In the second stage, for given values of the concession parameters, the container-terminal operators complete with each other to maximize their own profits by deciding the terminal handling charges in each container terminal. When a container terminal decides a terminal handling charge, it needs to consider the effects of the terminal handling charge on the rental fee to be paid to the port authority as well as the market share in the market. This study proposes and proves various properties of the optimal decision, which reduces the solution space significantly. Numerical experiments compare different rental fee schemes. Figure 1 gives an example of the two-stage game, in which there are one port authority and two container-terminal operators.

The remainder of this paper is organized as follows. Section 2 reviews previous studies. The revenue-sharing scheme with a single rate is introduced in Section 3. Section 4 discusses a revenue-sharing scheme with an incremental discount and that with an all-unit discount. The optimal

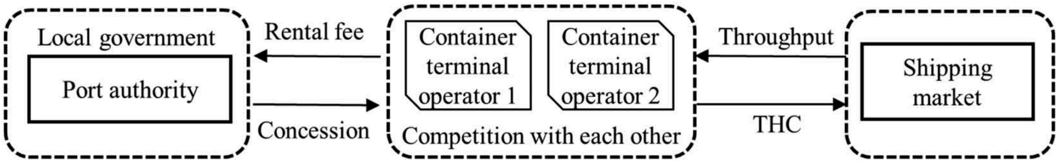

Figure 1. Structure of the two-stage game.

behaviors of the container-terminal operators for the concession contracts are analyzed. Methods for optimizing the parameters of the revenue-sharing scheme with a quantity discount are provided. The results of numerical experiments are given in Section 5. The conclusions and future studies are provided in Section 6.

2. Literature review

This section summarizes previous studies related to the terminal handling charge with a game theory point of view as well as previous studies related to optimizing the concession contracts.

2.1. Terminal handling charge and competition among container-terminal operators

The terminal handling charge is decided by a container-terminal operator and is charged to the shipping company for loading/unloading containers onto/from a vessel, which is an important parameter affecting the market share of a container-terminal operator. De Borger, Proost, and Van Dender (2008) proposed a two-stage game to examine the pricing behavior of ports and the investment policies in a port and the hinterland capacity. In the first stage, the local government decides the investment in the port capacity and hinterland connections to the port. In the second stage, the port determines the port prices considering the potential congestion and hinterland transport network.

Saeed and Larson (2010a, 2010b, 2013) studied a cooperative game among container-terminal operators in ports. In the first stage of the cooperative game, the container-terminal operators decide whether to act as a singleton or to enter into a coalition with other container-terminal operators. In the second stage, the cooperative container-terminal operators compete with other container-terminal operators that do not belong to their coalition by deciding the terminal handling charge. Bae et al. (2013) evaluated a non-cooperative two-stage game to investigate the container port competition for transshipment cargo in the duopoly market of transshipment ports and a continuum of identical shipping lines. In the first stage, each port decides its port price and in the second stage, the shipping lines simultaneously determine their calling ports while competing with each other. Xiao and Liu (2017) developed a two-stage oligopoly model to investigate container-hub port competition and cooperation in Northeast Asia considering the shippers, shipping lines, and ports. In the first stage of the oligopoly model, the ports decide their prices simultaneously to maximize their profits. In the second stage, the shipping lines allocate their container-handling quantities to the ports, and the shippers then assign their cargo to shipping liners. Dong (2018) proposed a theoretical model for a two-stage non-cooperative game to optimize the various pricing strategies between container terminals under deregulation.

2.2. Concession contracts

Based on a port concession contract, a port authority transfers the operation right of a terminal to a container-terminal operator. Setting the concession contract is a very complex process for the port authority and container-terminal operators. Many studies on the design of concession contracts addressed contracts based on revenue-sharing schemes. Most studies related to the design of concession contracts based on revenue sharing assumed that the revenue transferred to the port authority is dependent on the cargo throughput.

Chen and Liu (2014) proposed a two-stage game for maximizing the total revenue of a port authority by optimizing the parameters of the revenue-sharing scheme with a single rate. Chen and Liu (2015) used the same two-stage game model as that by Chen and Liu (2014) but with the objective of maximizing the traffic volume of the entire port instead of the total revenue of the port authority. Chen, Lin, and Liu (2017) extended Chen and Liu's (2014) study by assuming that terminal operators compete with each other using a terminal handling charge, instead of using the cargo amount. Liu et al. (2018) expanded Chen and Liu's (2015) study by assuming that each terminal has a constraint on the minimum throughput requirement. Han, Chen, and Liu (2018) also extended Chen and Liu (2014) study using different pursuing objectives, including the weighted sum of revenues and throughput benefits and social welfare. Variants of the revenue evaluation method have also been used, such as the percentage of cargo-handling charges collected by the container-terminal operators (Saeed and Larsen 2010a, 2010b, 2013) and the percentage of gross revenue collected by the container-terminal operators. Saeed and Larsen (2010b) examined the problem of designing a revenue-sharing concession contract assuming that the allocation of handling demand among terminals is accounted for by a multinomial logit demand model. They also used the rental fee with a single rate and attempted to optimize the single rate (unit fee).

Previous studies investigated revenue-sharing schemes extensively, in which the rental fee is evaluated using a single rate. Quantity discounts (Lee 1986) is a very popular method used by suppliers to encourage buyers to order in larger batches. Recently, Qiu and Lee (2019) first applied a quantity discount scheme to a dry port system and found that the quantity discount scheme can increase the profit of the dry port without adversely affecting shippers.

The contribution of this study may be summarized as follows. First, to the best of the authors' knowledge, this is the first theoretical study on the design of a revenue-sharing scheme with a quantity discount for a concession contract between a port authority and container-terminal operators. Second, the properties of the optimal solutions that are useful for solving the problem are proposed and proven. Third, this study shows that the revenue-sharing scheme is effective in increasing the cargo throughput in a port as well as for increasing the revenue of the port authority.

3. Revenue-sharing scheme with a single rate

This section introduces a revenue-sharing scheme with a single rate, which is the basis of the model in this study. Before providing details of the concession, the assumptions and notations used in this study are first introduced.

The assumptions of the models in this study are as follows:

- This study assumes a port where there are one port authority and two container-terminal operators competing with each other.
- The port authority grants the same concession to the two container-terminal operators.
- The concession adopted by the port authority is the revenue-sharing scheme, in which the rental fee is proportional to the yearly throughput (TEU).
- The objective of the port authority is to maximize the total revenue collected from the two container-terminal operators by determining the optimal parameters in the concession contract.
- The two container-terminal operators compete with each other for maximizing their profit by optimally deciding the terminal handling charge considering the concession contract with the port authority.
- The cargo throughputs of terminals and the terminal handling charges per TEU of terminals follow a linear relationship, which was proposed by Chen and Liu (2014).

The parameters and decision variables are as follows:

Parameters

i : Index for a container-terminal operator.
b : Service substitution parameter. $b \in (0, 1)$.
c_i : Variable cost for handling one TEU by container-terminal operator i; c_2 was assumed to be greater than c_1.

Decision variables

p_i : Terminal handling charge per TEU of container-terminal operator i.
R : Rental fee per TEU charged by the port authority to container-terminal operators.

Dependent variables

q_i : Annual throughput (TEU) handled by container-terminal operator i. This is dependent on p_i (i = 1, 2).

Chen and Liu (2014) proposed a game model for two competitive container-terminal operators with the relationship between the amount of cargo and the terminal handling charge as follows:

$$p_1 = 1 - q_1 - bq_2 \tag{1}$$

$$p_2 = 1 - q_2 - bq_1 \tag{2}$$

Representing q_1 and q_2 as functions of p_1 and p_2, $q_1 = \frac{1}{1+b} + \frac{bp_2 - p_1}{1-b^2}$ and $q_2 = \frac{1}{1+b} + \frac{bp_1 - p_2}{1-b^2}$. The profit of each container-terminal operator then becomes $\pi_i(R) = p_i q_i - (c_i + R)q_i$. Chen and Liu (2017) obtained the Nash equilibrium (NE) analytically for the competitive game as follows:

$$p_1^* = \frac{1-b+R}{2-b} + \frac{2c_1+bc_2}{4-b^2} \text{ and } p_2^* = \frac{1-b+R}{2-b} + \frac{2c_2+bc_1}{4-b^2}.$$

After obtaining the optimal p_1^* and p_2^*, the optimal annual throughput, q_1^* and q_2^*, can be derived as follows:

$$q_1^* = \frac{1-R}{(1+b)(2-b)} + \frac{\mathrm{b}c_2 + b^2c_1 - 2c_1}{(1-b^2)(4-b^2)} \text{ and } q_2^* = \frac{1-R}{(1+b)(2-b)} + \frac{\mathrm{b}c_1 + b^2c_2 - 2c_2}{(1-b^2)(4-b^2)} \tag{3}$$

The profit becomes

$$\pi_i^*(R) = \left(q_i^*\right)^2 \text{ for } i = 1, 2. \tag{4}$$

The revenue of the port authority can be expressed as.

$$Z(R) = R(q_1 + q_2) \tag{5}$$

The optimal rental fee per TEU may be derived (Chen, Lin, and Liu 2017) as follows:.

$$R^* = \frac{1}{2} - \frac{c_1 + c_2}{4}$$

4. Revenue-sharing scheme with a quantity discount

When the amount of cargo of a container-terminal operator increases by active promotion and marketing activities and the container-terminal operator has a contract on its concession fee that is proportional to the amount of cargo, even though the revenue of the port authority increases, this will increase the rental cost burden to the container-terminal operator and discourage the container-terminal operator to promote sales. To motivate the container-terminal operators to boost their container throughput, the port authority may provide a discount on the unit rate of the rental fee when the throughput exceeds a discount breakpoint. This section discusses how container-terminal operators behave for a given concession fee scheme with a quantity discount and how the port authority can design the concession parameters to maximize its own revenue.

This section introduces two revenue-sharing schemes with a quantity discount: an incremental discount and an all-unit discount. In revenue sharing with a quantity discount, the relationship between the unit container fee charged by the port authority and the container throughput is usually a decreasing piecewise function. Lee and Lee (2012) introduced a revenue-sharing scheme with a quantity discount utilized by the Busan Port Authority.

In a revenue-sharing scheme with an all-unit discount, the container-terminal operator will be charged with the discount for all containers only if the annual throughput exceeds the discount breakpoint, whereas in an incremental quantity discount, the discounted rate is applied only to the annual throughput exceeding the discount breakpoint.

The following decision variables will be used to represent the rental fee per unit under the quantity discount schemes:

Decision variables

R_1 : Rental fee per unit without a discount
R_2 : Rental fee per unit with a discount
Q : Rental fee breakpoint beyond which the marginal rental fee becomes R_2

Figure 2 gives an example of revenue sharing with an incremental discount and that with an all-unit discount; the price breakpoint equals 0.5.

In the revenue-sharing scheme with an incremental discount (refer to Figure 2(a)), the annual rental fee becomes

$$r_i(q_i) = \begin{cases} R_1 q_i, & \text{when } q_i < Q \\ R_1 Q + R_2(q_i - Q), & \text{when } q_i \geq Q, \end{cases} \tag{6}$$

while in the revenue-sharing scheme with an all-unit discount (refer to Figure 2(b)),

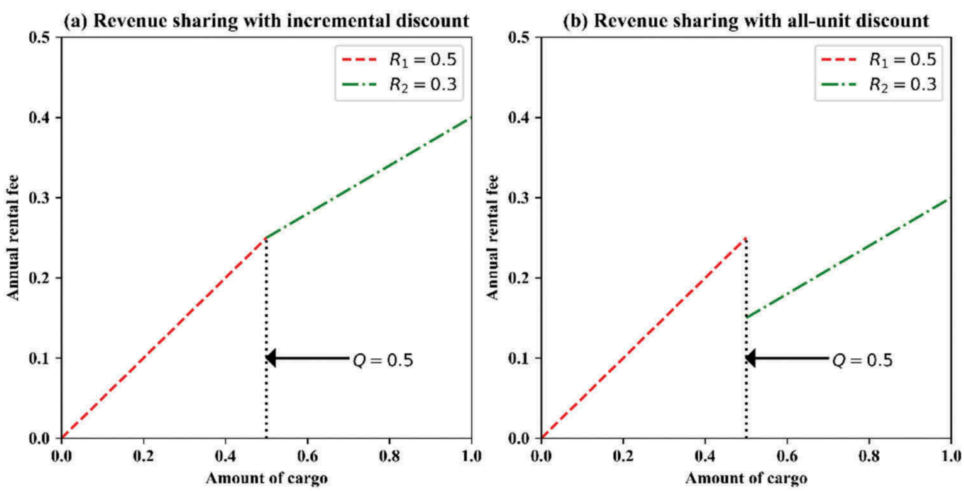

Figure 2. Illustrative rental fee for an incremental and an all-unit quantity discount.

$$r_i(q_i) = \begin{cases} R_1 q_i, & \text{when } q_i < Q \\ R_2 q_i, & \text{when } q_i \geq Q, \end{cases} \tag{7}$$

where $R_1 \geq R_2$.

The difference between revenue sharing with an incremental discount scheme and that with an all-unit discount scheme is that the former provides a discount only for the additional amount of cargo over Q, whereas the latter provides a discount for all cargo if the amount of cargo exceeds Q. Details of revenue sharing with an incremental discount scheme and that with an all-unit discount scheme are introduced in the following.

4.1. Revenue-sharing scheme with an incremental discount

4.1.1. Optimal behaviors of container-terminal operators

The profit function for the container-terminal operator i may be expressed as follows:

$$\pi_i = \begin{cases} p_i q_i - (c_i + R_1) q_i & q_i < Q \\ p_i q_i - c_i q_i - R_1 Q - R_2 (q_i - Q) & q_i \geq Q. \end{cases} \tag{8}$$

Given (R_1, R_2, Q), which is provided by the port authority, the container-terminal operators compete with each other by optimizing their own handling prices (p_1^*, p_2^*), which satisfy the first-order necessary conditions for maximizing π_i, $\frac{\partial \pi_1}{\partial p_1} = \frac{\partial \pi_2}{\partial p_2} = 0$. The formulae of $\frac{\partial \pi_1}{\partial p_1}$ and $\frac{\partial \pi_2}{\partial p_2}$ are provided in the supplemental material (https://ieyjzhou.github.io/files/Supplemental_Material_MPM_2019.pdf). Hence, $\frac{\partial^2 \pi_i}{\partial (p_i)^2} = \frac{-2}{1-b^2}$ and $\frac{\partial^2 \pi_i}{\partial p_1 p_2} = \frac{b}{1-b^2}$. Thus, the Hessian matrix, H >0, implies that π_i is convex with respect to p_1 and p_2. When the equations $\frac{\partial \pi_1}{\partial p_1} = 0$ and $\frac{\partial \pi_2}{\partial p_2} = 0$ are solved simultaneously, the NE solution of the cargo amounts (p_1^*, p_2^*) can be obtained. The optimal (q_1^*, q_2^*) can be derived easily based on the relationship function between p_i and q_i.

Figure 3 shows the changes in the optimal quantities when Q decreases from a large value for given values of R_1 and R_2. When the value of Q is large, both container-terminal operators 1 and 2 are not entitled to the discount (Case 1: Figure 3(1)). Let the optimal quantities of terminal operators 1 and 2 be q_{11}^* and q_{21}^*, respectively, in Case 1. When Q becomes smaller and reaches Q_1, at which π_1 with a discount and that without a discount become the same (The expression of Q_1 is provided in Appendix A), container-terminal operator 1 is entitled to the discount but container-terminal operator 2 is not, which is called Case 2. The optimal quantity of container-terminal operator 1 and container-terminal operator 2 will be denoted as q_{12}^* and q_{22}^*, respectively, in Case 2. When Q becomes smaller and reaches Q_2, at which π_2 with a discount and that without a discount become the same (Appendix A gives the expression of Q_2), container-terminal operator 2 also becomes entitled to the discount, which is called Case 3. Note that container-terminal operator 1 becomes entitled to the discount before container-terminal operator 2 $(Q_1 > Q_2)$, which can be easily proven. The optimal quantity of container-terminal operator 1 and container-terminal operator 2 will be denoted as q_{13}^* and q_{23}^*, respectively, in Case 3.

Let $q_i(x_1, x_2, q_1, q_2)$ and $\pi_i(x_1, x_2, q_1, q_2)$ be the annual throughput and the profit of terminal operator i when the marginal rental fee per unit and the annual throughput for terminal operator i is x_i and q_i, respectively. By solving the first-order necessary conditions for cases 1, 2, and 3, the optimal cargo amount (q_1^*, q_2^*) and optimal profit $\left(\pi_1^*, \pi_2^*\right)$ can be obtained, as shown in Table 1.

Property 1: For any given arbitrary R_1, R_2, and Q, the following relationships hold:

$$q_{22}^* < q_{21}^* < q_{11}^* < Q < q_{23}^* < q_{13}^* < q_{12}^*.$$

Proof: The proof is straightforward and is included in the supplemental material.

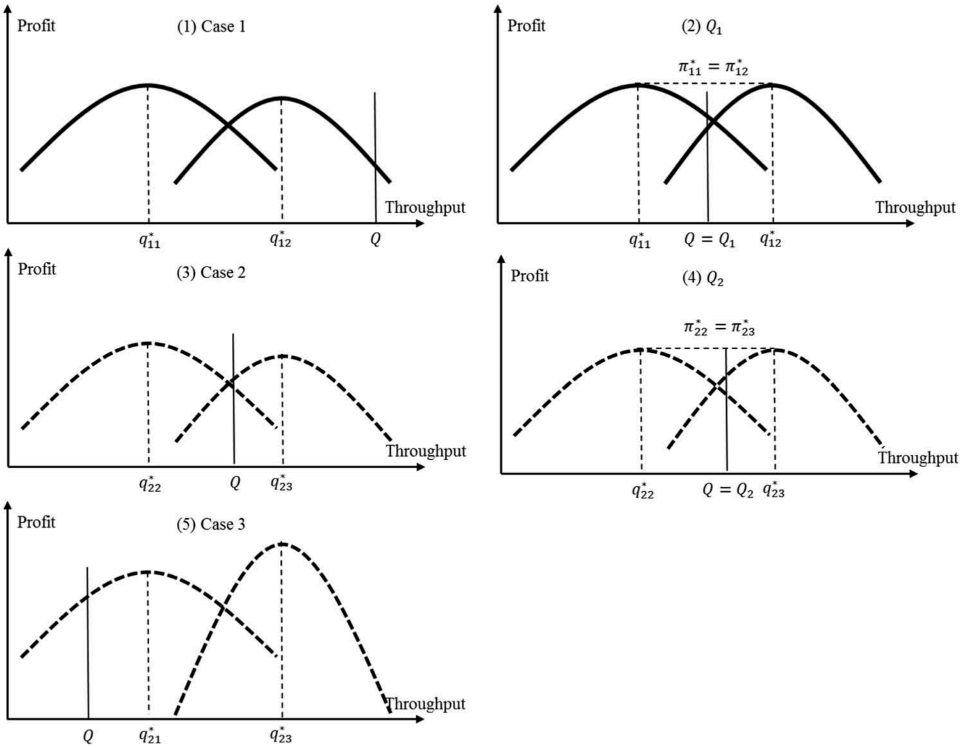

Figure 3. Changes in the optimal throughput for various values of Q (dotted curve: profit function of container-terminal operator 2; solid curve: that of container-terminal operator 1).

4.1.2. Optimal concession contracts

The port authority will attempt to determine the parameters of the rental fee in a way of maximizing its revenue from two container-terminal operators:

$$Z(Q, R_1, R_2) = r_1(q_1) + r_2(q_2) \tag{9}$$

Property 2: For the revenue-maximizing problem of the port authority under the rental fee scheme with an incremental discount, Case1 is dominated by Case 3.

Proof: The proof is provided in Appendix B.

Property 2 implies that the optimal solution of Q may exist only in Case 2 or Case 3. That is, Case 1 does not need to be considered when finding the optimal solution. Based on Property 2, Property 3 provides a simple way of finding the optimal value of Q for a given (R_1, R_2).

Property 3: For a given (R_1, R_2), the optimal value of Q is Q_1 in Case 2, whereas it is Q_2 in Case 3. To find the optimal parameters of the rental fee scheme with the incremental discount, it is sufficient to find the optimal R_1 and R_2 while maximizing $Max\{Z(Q_1), Z(Q_2)\}$.

Proof: The proof is provided in Appendix C.

Because, for a given set of R_1 and R_2 values, the optimal Q value may be found by comparing only two values of Q, an enumeration search algorithm may be applied to find the optimal values of R_1, R_2. This study enumerates the feasible range of R_1 and R_2 using small step sizes in the numerical experiments of Section 5.

Table 1. Optimal cargo amounts (q_1^*, q_2^*) and optimal profit (π_1^*, π_2^*) for container-terminal operators 1 and 2 for the revenue-sharing scheme with an incremental discount.

Cases	q_1^*, q_2^*	π_1^*, π_2^*
1	$q_1^*(R_1, R_1, q_1, q_{21}^*) = q_{11}^* = \frac{1-R_1}{(1+b)(2-b)} + \frac{bc_2 - c_1(2-b^2)}{(1-b^2)(4-b^2)}$ $q_2^*(R_1, R_1, q_{11}^*, q_2) = q_{21}^* = \frac{1-R_1}{(1+b)(2-b)} + \frac{bc_1 - c_2(2-b^2)}{(1-b^2)(4-b^2)}$	$\pi_1^*(R_1, R_1, q_{11}^*, q_{21}^*) = \pi_{11}^* = (1-b^2)(q_{11}^*)^2$ $\pi_2^*(R_1, R_1, q_{11}^*, q_{21}^*) = \pi_{21}^* = (1-b^2)(q_{21}^*)^2$
2	$q_1^*(R_2, R_1, q_1, q_{22}^*) = q_{12}^* = \frac{1}{(1+b)(2-b)} + \frac{(b^2-2)(c_1+R_2)+b(c_2+R_1)}{(4-b^2)(1-b^2)}$ $q_2^*(R_2, R_1, q_{12}^*, q_2) = q_{22}^* = \frac{1}{(1+b)(2-b)} + \frac{(b^2-2)(c_2+R_1)+b(c_1+R_2)}{(4-b^2)(1-b^2)}$	$\pi_1^*(R_2, R_1, q_{12}^*, q_{22}^*) = \pi_{12}^* = (1-b^2)(q_{12}^*)^2 + R_2Q - R_1Q$ $\pi_2^*(R_2, R_1, q_{12}^*, q_{22}^*) = \pi_{22}^* = (1-b^2)(q_{22}^*)^2$
3	$q_1^*(R_2, R_2, q_1, q_{23}^*) = q_{13}^* = \frac{1-R_2}{(1+b)(2-b)} + \frac{bc_2 - c_1(2-b^2)}{(1-b^2)(4-b^2)}$ $q_2^*(R_2, R_2, q_{13}^*, q_2) = q_{23}^* = \frac{1-R_2}{(1+b)(2-b)} + \frac{bc_1 - c_2(2-b^2)}{(1-b^2)(4-b^2)}$	$\pi_1^*(R_2, R_2, q_{13}^*, q_{23}^*) = \pi_{13}^* = (1-b^2)(q_{13}^*)^2 + R_2Q - R_1Q\pi_2^*(R_2, R_2, q_{13}^*, q_{23}^*) = \pi_{23}^* = (1-b^2)(q_{23}^*)^2 + R_2Q - R_1Q$

Property 4: The optimal solution of the port authority is obtained from Case 3; container-terminal operator 2 cannot receive any positive profit.

Proof: The proof is provided in Appendix D.

Property 4 says that container-terminal operators cannot obtain any positive profit if the port authority optimizes its decision, and as a result, container-terminal operators become Case 3. According to the experiment results in Section 5, the profit of container-terminal operator 2 becomes zero in the schemes with an incremental discount or with an all-unit discount. On the other hand, this is proved analytically only for the scheme with an incremental discount. The revenue of the port authority is increased as the cost of the profits of the container-terminal operators until the profit of the less competitive container-terminal operator vanishes.

Property 5: The total throughput of two container-terminal operators is maximized when $R_1 = R_2 = 0$.

Proof: The proof is provided in Appendix E.

Property 5 says that if the port authority wants to maximize the total throughput of all the container-terminal operators instead of the total revenue of the port authority, it would be better not to receive any rental fee from the container-terminal operators.

Property 6: The optimal revenue-sharing scheme with an incremental discount does not give a lower revenue than that without discount.

Proof: The proof is provided in Appendix F.

Property 6, together with Property 11, implies that the schemes with a discount outperform those without discount in terms of the revenue of the port authority. This is because the former has more control variables for inducing the container-terminal operators to increase their throughputs than the latter.

4.2. Revenue-sharing scheme with an all-unit discount

4.2.1. Optimal behaviors of container-terminal operators

The profit function for container-terminal operator i can be expressed as follows:

$$\pi_i = \begin{cases} p_i q_i - (c_i + R_1) q_i & q_i < Q \\ p_i q_i - (c_i + R_2) q_i & q_i \geq Q. \end{cases} \tag{10}$$

Figure 4 shows two profit functions: one with the normal rate, R_1; and one with the discounted rate, R_2. When $Q > q_{i(0)}$, the container-terminal operators will choose $q_{i(1)}$. When $Q < q_{i(0)}$, the container-terminal operators will choose $q_{i(2)}$. When $q_{i(2)} \leq Q \leq q_{i(0)}$, the container-terminal operators will choose Q. Therefore, there are three possible values of q_i^*: $q_{i(1)}$, $q_{i(2)}$, and Q. That is, each container-terminal operator chooses one of the above three values for any value of q_i of the opponent. For two players, there are nine combinations of (q_1, q_2). If the cases are described by (u_1, u_2, q_1, q_2), where u_i represents the rental fee per unit of container-terminal operator i, they are (Case 1) $\left(R_1, R_1, q_{1(1)}, q_{2(1)}\right)$; (Case 2) $\left(R_2, R_2, q_{1(2)}, q_{2(2)}\right)$; (Case 3) $\left(R_2, R_1, q_{1(2)}, q_{2(1)}\right)$; (Case 4) $\left(R_2, R_1, Q, q_{2(1)}\right)$; (Case 5) $\left(R_2, R_2, q_{1(2)}, Q\right)$; (Case 6) (R_2, R_2, Q, Q); (Case 7) $\left(R_1, R_2, q_{1(1)}, q_{2(2)}\right)$; (Case 8) $(R_1, R_2, q_{1(1)}, Q)$; (Case 9) $\left(R_2, R_2, Q, q_{2(2)}\right)$.

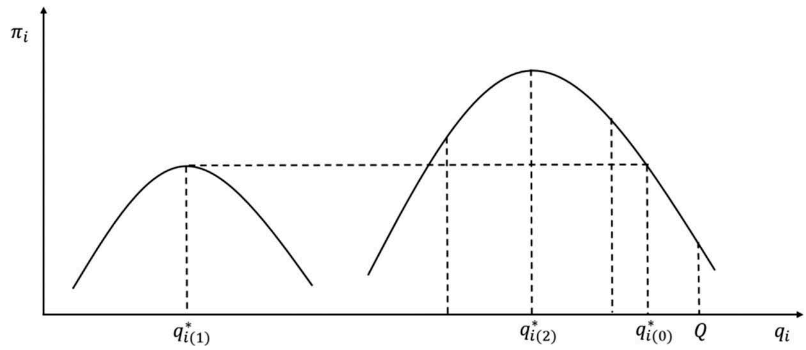

Figure 4. Profit function of a container-terminal operator.

Property 7: Cases 7, 8, and 9 cannot occur.

Proof: See Appendix G.

Property 7 simplifies the solution space by removing the three cases. Figure 5 classifies the cases based on the relative positions of Q. The NE quantity can be derived easily using the first-order necessary conditions, as listed in Table 2.

Figure 5 shows various cases of optimal throughputs. In Figure 5, the dashed curve denotes the profit function for container-terminal operator 1 and the solid curve represents the profit function for container-terminal operator 2.

From $c_2 > c_1$, $R_2 < R_1$, and $0 < b < 1$, the following inequalities follow:

$q_{11} < q_{12} < q_{13}, q_{23} < q_{21} < q_{22}, q_{11} > q_{21}, q_{12} > q_{22}$.

The values of various Q_i may be derived as follows. Q_1 is the value of Q satisfying $\pi_{13}\left(R_2, R_1, \ Q, q'_{23}\right) = \pi_{11}(R_1, R_1, \ q_{11}, q_{21})$ and Q_2 is the value of q_{13}. Q_3 is the value of Q satisfying $\pi_{22}(R_2, R_2, \ Q, Q) = \pi_{23}\left(R_2, R_1, \ Q, q'_{23}\right)$. Q_4 is the value of Q satisfying $\pi_{22}(R_2, R_2, \ q_{12}, Q) = \pi_{23}(R_2, R_1, \ q_{13}, q_{23})$. By solving the above equations of Q, Q_1, Q_2, Q_3, and Q_4 can be derived, which are presented in Appendix H.

4.2.2. Optimal concession contracts

This section derives useful properties of the optimal parameters of the rental fee and proposes an efficient algorithm for finding the optimal solution based on the derived properties.

Property 8: For the revenue maximizing problem of the port authority with an all-unit discount, for a given (R_1, R_2), the optimal solutions of Cases 1 and 2 are dominated by that of Case 3 and the optimal solution of Case 3 is dominated by those of Cases 4 and 5.

Proof: The proof is provided in Appendix I.

Property 8 implies that the optimal solution may be found in Cases 4, 5, or 6, which reduces the solution space further.

Property 9: To find the optimal concession contract, it is sufficient to find the optimal R_1 and R_2 by maximizing $Z(Q)$ when $Z(Q) = Max\ \{Z(Q_1), Z(Q_3), Z(Q_4)\}$ if $R_2 \geq bR_1/2$; otherwise $Z(Q) = Max\{Z(Q_2), Z(Q_3), Z(Q_4)\}$.

Proof: The proof is provided in Appendix J.

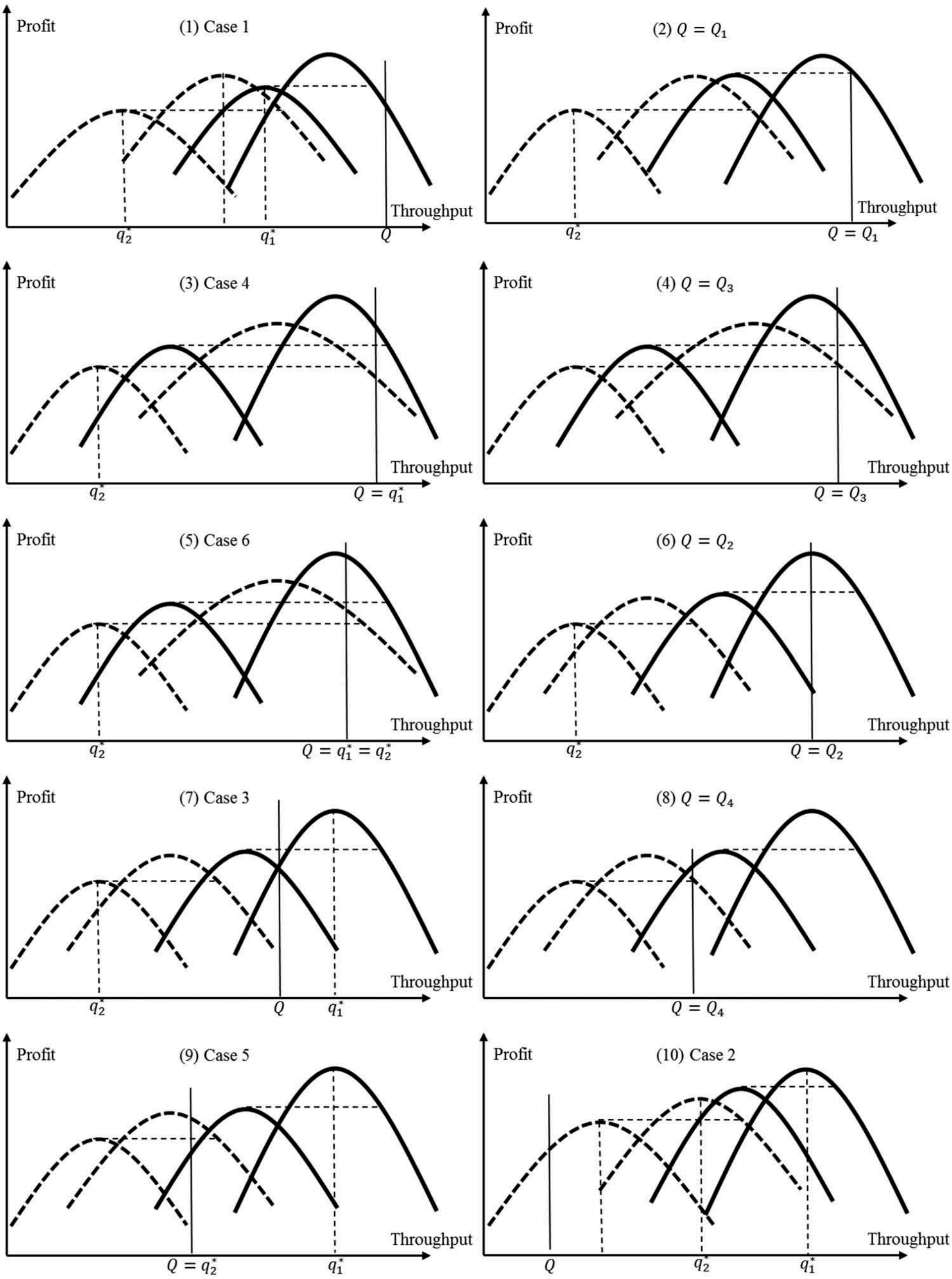

Figure 5. Various cases of optimal throughputs (dotted curve: profit function of container-terminal operator 2; solid curve: that of container-terminal operator1).

By Property 9, for a given R_1 and R_2, the optimal value of Q can be found by simply comparing three discrete values of Q, which reduces the search space significantly.

Property 10: The total throughput of two container-terminal operators is maximized when $R_1 = R_2 = 0$.

Table 2. Optimal cargo amounts (q_1^*, q_2^*) and optimal profit (π_1^*, π_2^*) for container-terminal operators 1 and 2 for a revenue-sharing scheme with all-unit discount.

Case	Simplified notation	Formal notation	Expression
1	q_{11}	$q_1^*(R_1, R_1,\ q_1, q_{21})$	$\frac{1-R_1}{(1+b)(2-b)} + \frac{bc_2 - c_1(2-b^2)}{(1-b^2)(4-b^2)}$
	q_{21}	$q_2^*(R_1, R_1,\ q_{11}, q_2)$	$\frac{1-R_1}{(1+b)(2-b)} + \frac{bc_1 - c_2(2-b^2)}{(1-b^2)(4-b^2)}$
2	q_{12}	$q_1^*(R_2, R_2,\ q_1, q_{22})$	$\frac{1-R_2}{(1+b)(2-b)} + \frac{bc_2 - c_1(2-b^2)}{(1-b^2)(4-b^2)}$
	q_{22}	$q_2^*(R_2, R_2,\ q_{12}, q_2)$	$\frac{1-R_2}{(1+b)(2-b)} + \frac{bc_1 - c_2(2-b^2)}{(1-b^2)(4-b^2)}$
3	q_{13}	$q_1^*(R_2, R_1,\ q_1, q_{23})$	$\frac{1}{(2-b)(1+b)} + \frac{b(c_2+R_1)+(b^2-2)(c_1+R_2)}{(4-b^2)(1-b^2)}$
	q_{23}	$q_2^*(R_2, R_1,\ q_{13}, q_2)$	$\frac{1}{(2-b)(1+b)} + \frac{b(c_1+R_2)+(b^2-2)(c_2+R_1)}{(4-b^2)(1-b^2)}$
4	$q_{13}^{'}$	$q_1^*(R_2, R_1,\ Q, q_{23}^{'})$	Q
	$q_{23}^{'}$	$q_2^*(R_2, R_1,\ Q, q_2)$	$\frac{1-bQ-c_2-R_1}{2}$
5	$q_{12}^{'}$	$q_1^*(R_2, R_2,\ q_1, Q)$	$\frac{1-bQ-c_1-R_2}{2}$
	$q_{22}^{'}$	$q_2^*(R_2, R_2,\ q_{12}^{'}, Q)$	Q
6	$q_{12}^{''}$	$q_1^*(R_2, R_2,\ Q, Q)$	Q
	$q_{22}^{''}$	$q_2^*(R_2, R_2,\ Q, Q)$	Q

Proof: The proof is provided in Appendix K.

Property 11: The optimal revenue-sharing scheme with an all-unit discount does not give a lower revenue than that without discount.

Proof: The proof is the same as that for Property 6.

A search algorithm for finding the optimal values of R_1, R_2, and Q is similar to the search algorithm for a revenue-sharing scheme with an incremental discount in Section 4.1.2 except that the candidates for the optimal Q are different from those of the scheme with an incremental discount.

5. Numerical examples

This section provides numerical examples to analyze the proposed two-stage game.

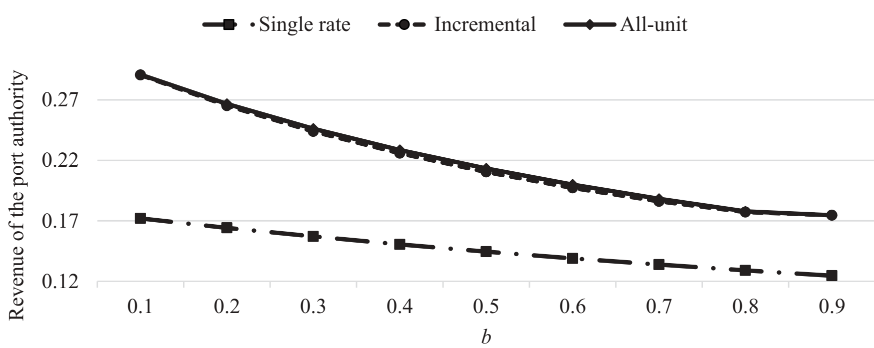

Figure 6. Comparison of the revenue of the port authority for various values of b.

5.1. Comparison of various revenue-sharing schemes and sensitivity analysis

Two groups of experiments were performed to see how the parameters, including c_1, c_2, and b, affect the revenue of the port authority. In the first group of experiments, the parameters, c_1 and c_2, were fixed to 0.1 and 0.2, respectively. The value of b ranged from 0.1 to 0.9. Three revenue-sharing schemes were compared: revenue sharing with a single rate, revenue sharing with an incremental discount, and that with all-unit discount.

Figure 6 compares the revenue of the port authority for various values of b. Note that as the value of b becomes larger, the effect of the quantity of one company on the price of the other company becomes higher. The revenue of the port authority decreases with increasing b for all three schemes. The single rate scheme showed the lowest revenue. This is because the schemes with a discount have more control variables than that with a single rate for inducing container-terminal operators to increase their throughputs. Among the three revenue-sharing schemes, the revenue by the scheme with an all-unit discount was the highest in almost all the experimental results. Note that the scheme with an all-unit discount provides a discount to the entire annual throughput if the throughput exceeds a price breakpoint, while that with an incremental discount provides a discount only to the amount exceeding the price breakpoint. Thus, the scheme with an all-unit discount has a direct and stronger inducing power than that with an incremental discount, which is why the scheme with an all-unit discount provides a higher revenue than that with an incremental discount. On the other hand, the higher revenue of the port authority comes from the reduction in the profit of container-terminal operator 1.

Table 3 lists the change in the optimal solution for various values of b. For a revenue-sharing scheme with a single rate, the optimal R^* remains constant for various values of b, which is $\frac{1}{2} - \frac{c_1+c_2}{4}$. With increasing b, the optimal q_1^* and q_1^* decreases for the revenue-sharing schemes with a single rate. The optimal R_1 decreases with increasing b for both revenue-sharing schemes with a discount. The optimal q_1^* and q_2^* always decreases with increasing b when the NE solution comes from the same case for both revenue-sharing schemes with a discount. This is because, for a larger value of b, the effect of the throughput of a container-terminal operator on the price of the other operator becomes larger, which forces the throughput at NE to remain at a lower level. As a result, the revenue of the port authority becomes lower. In both discount schemes, R_1^* decreases with increasing b. On the other hand, R_2^* does not show a consistent trend compared to that of R_1^* for the change in b.

Table 4 compares the profits of container-terminal operators. The profit of container-terminal operator 2 is always zero for both revenue-sharing schemes with a discount. The port authority maximizes its revenue at the cost of the profit of the terminals and container-terminal operator 2, which is less competitive because of the higher operation cost, has zero profit. The profit of

Table 3. Optimal decisions in various concession schemes and values of b.

		b								
Schemes	Solutions	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Single rate	R^*	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425
	q_1^*	0.2287	0.2210	0.2142	0.2083	0.2033	0.1992	0.1959	0.1935	0.1920
	q_2^*	0.1761	0.1654	0.1554	0.1458	0.1367	0.1277	0.1189	0.1101	0.1011
Incremental	Q^*	0.1678	0.1502	0.1339	0.1189	0.1044	0.0896	0.0733	0.0524	0.2979
	R_1^*	0.759	0.721	0.686	0.651	0.616	0.579	0.539	0.493	0.421
	R_2^*	0.092	0.134	0.176	0.217	0.257	0.298	0.338	0.379	0.399
	q_1^*	0.3891	0.3599	0.3360	0.3173	0.3036	0.2945	0.2934	0.3073	0.4209
	q_2^*	0.3362	0.3031	0.2739	0.2479	0.2236	0.1984	0.1699	0.1288	0.0002
	Case	3	3	3	3	3	3	3	3	2
All-unit	Q^*	0.3636	0.3333	0.3077	0.2857	0.2667	0.2500	0.2353	0.2222	0.4165
	R_1^*	0.764	0.733	0.708	0.686	0.667	0.65	0.635	0.622	0.425
	R_2^*	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.419
	q_1^*	0.3636	0.3333	0.3077	0.2857	0.2667	0.2500	0.2353	0.2222	0.4165
	q_2^*	0.3636	0.3333	0.3077	0.2857	0.2667	0.2500	0.2353	0.2222	0.0001
	Case[a]	6	6	6	6	6	6	6	6	4

[rma]Cases are explained in Tables 1 and 2.

Table 4. Comparison of the profits of container-terminal operators for various values of b.

	Container-terminal operator 1			Container-terminal operator 2	
b	Single rate	Incremental	All-unit	Single rate	Incremental/all-unit
0.1	0.0523	0.0380	0.0364	0.0310	0.0000
0.2	0.0488	0.0362	0.0333	0.0274	0.0000
0.3	0.0459	0.0345	0.0308	0.0241	0.0000
0.4	0.0434	0.0330	0.0286	0.0213	0.0000
0.5	0.0413	0.0316	0.0267	0.0187	0.0000
0.6	0.0397	0.0303	0.0250	0.0163	0.0000
0.7	0.0384	0.0292	0.0235	0.0141	0.0000
0.8	0.0374	0.0280	0.0222	0.0121	0.0000
0.9	0.0369	0.0271	0.0268	0.0102	0.0000

Table 5. Optimal decisions in various concession schemes for various values of c_2/c_1.

		c_2/c_1								
Schemes	Solutions	29.00	14.00	9.00	6.50	5.00	4.00	3.29	2.75	2.33
Single rate	R^*	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425
	q_1^*	0.2633	0.2567	0.25	0.2433	0.2367	0.23	0.2233	0.2167	0.21
	q_2^*	0.0767	0.0833	0.09	0.0967	0.1033	0.11	0.1167	0.1233	0.13
Incremental	Q^*	0.3378	0.3297	0.3226	0.3155	0.3080	0.0820	0.0877	0.0933	0.0986
	R_1^*	0.4850	0.4970	0.5080	0.5100	0.5100	0.5750	0.5850	0.5950	0.6060
	R_2^*	0.2030	0.2000	0.1940	0.1920	0.1920	0.2930	0.2840	0.2750	0.2670
	q_1^*	0.4497	0.4457	0.4434	0.4370	0.4290	0.3196	0.3156	0.3116	0.3071
	q_2^*	0.0001	0.0001	0.0002	0.0066	0.0146	0.1756	0.1876	0.1996	0.2111
	Case[a]	2	2	2	2	2	3	3	3	3
All-unit	Q^*	0.4517	0.4475	0.4448	0.4408	0.2500	0.2533	0.2567	0.2600	0.2633
	R_1^*	0.4840	0.4960	0.5070	0.5190	0.6240	0.6330	0.6410	0.6490	0.6580
	R_2^*	0.4130	0.4180	0.4210	0.4250	0.3750	0.3800	0.3850	0.3900	0.3950
	q_1^*	0.4517	0.4475	0.4448	0.4408	0.2500	0.2533	0.2567	0.2600	0.2633
	q_2^*	0.0001	0.0001	0.0003	0.0003	0.2500	0.2533	0.2567	0.2600	0.2633
	Case[a]	4	4	4	4	6	6	6	6	6

[a]Cases are explained in Tables 1 and 2.

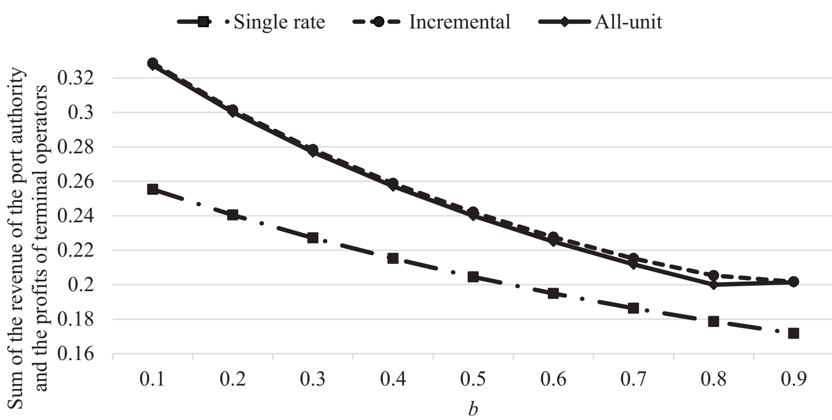

Figure 7. Comparison of the sum of the revenue of the port authority and the profits of container-terminal operators for various values of b.

container-terminal operator 1 in the all-unit discount is lower than that in the incremental discount scheme. Considering the results in Tables 3 and 5 together, the all-unit discount scheme provides

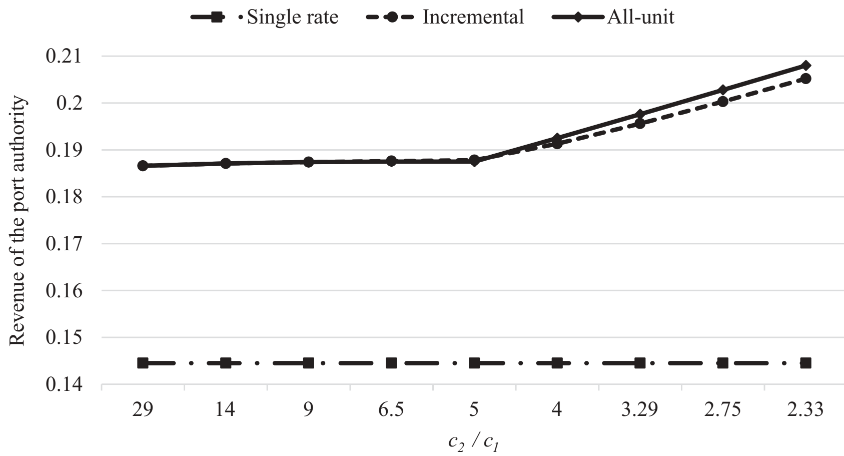

Figure 8. Comparison of the revenue of the port authority for various values of c_2/c_1.

higher revenue to the port authority than the incremental discount scheme at the cost of the profit of container-terminal operator 1.

Figure 7 compares the sum of the revenue of the port authority and the profits of the container-terminal operators, which may be regarded as the total benefit of the system. The revenue-sharing scheme with the incremental discount had the highest total benefit of the system among the three schemes. The gaps in the total benefit of the system between the revenue-sharing scheme with an incremental discount and that with an all-unit discount were very small, ranging from 0.14% to 2.63%.

In the second group of experiments, the ratio of c_2/c_1 was changed while maintaining $c_2 + c_1 = 0.3$. As shown in Figure 8, in the revenue-sharing scheme with a single rate, the ratio of c_2/c_1 does not affect R^* and the revenue of the port authority because R^* and the revenue are functions of $c_2 + c_1$, which is assumed to be fixed. On the other hand, in the revenue-sharing scheme with a discount, the revenue of the port authority increases with a decreasing c_2/c_1 ratio. When two container-terminal operators are too different from each other, the rental fee scheme with a discount cannot induce both container-terminal operators but has to focus on either of two operators. When they are similar in their profit functions, however, the discount policy can induce both container-terminal operators to increase their throughputs at the same time.

Table 5 lists the changes in the optimal solutions for various ratios of c_2/c_1. q_1^* decreases with decreasing c_2/c_1, while q_2^* increases and q_1^* decreases for all three schemes, as expected. As c_2/c_1 decreases from 29.00 to 2.33, R_1^* and R_2^* in the scheme with an all-unit discount increases within the same case.

5.2. Managerial implications and additional discussions about the models

From the numerical experiments, the following important managerial insights could be obtained. First, the revenue-sharing schemes with a discount can increase the revenue of the port authority significantly compared to the revenue-sharing scheme with a single rate. The numerical experiment also showed that the revenue-sharing scheme with an all-unit discount scheme provides higher revenue to the port authority than that with an incremental discount in almost all the experiment results. From the viewpoint of the port authority that attempts to maximize the total revenue, the revenue-sharing schemes with a discount must be useful. When designing detail concession contracts at real practices, however, the port authority should consider that a part of the revenue increase comes from the decrease in profits of the container-terminal operators. The revenue-sharing scheme with an all-unit discount has a stronger power of inducing container-terminal operators to increase their throughputs than that with an incremental discount.

The sum of the revenue of the port authority and the profits of the container-terminal operator were also increased by applying the revenue-sharing schemes with a discount. This result is meaningful in that although the port authority attempts to maximize its revenue, the schemes with a discount contribute significantly to the improvement of the utility of the entire system compared to the scheme with a single rate.

As the service substitution parameter becomes smaller, which means that the influence of the throughput of one container-terminal operator on the price of the other operator becomes smaller, and the operation cost parameters of the two container-terminal operators become similar, the revenue of the port authority becomes larger for all three revenue-sharing schemes.

If the port authority aims to maximize the total throughput, the optimal rental fee always equals zero. This means that for maximizing the total throughput of a port, the best policy is not to receive any rental fee that is based on the annual throughput. A fixed annual fee may be collected. According to the experiment results, the revenue-sharing schemes with a discount increase the total container throughput significantly compared to that with a single rate (see Figure 9). This means that when a port authority wants to increase the total throughput of a port, the revenue-sharing schemes with a discount can be an effective alternative as a rental fee scheme.

In the game theory model, there are two competition models: the Cournot (quantity competition) and Bertrand (price competition) models, in which companies consider the quantity or price of a product as a decision variable to compete with each other (Darrough 1993). This study assumed that the container-terminal operators are competing through the terminal handling charge (Bertrand competition). With the same values of the parameters used in the previous subsection, the Bertrand and Cournot competition models were compared by varying the values of b and c_2/c_1, respectively. The value of b was varied between 0.1 and 0.9, while the value of c_2/c_1 was varied between 2.33 and 29.00 with the condition, $c_2 + c_1 = 0.3$. The price competition model always gave higher revenues to the port authority than those by the quantity competition model. When the value of b was varied, the average gap was 1.96% and 0.31% for the scheme with an incremental and an all-unit discount, respectively. When the value of c_2/c_1 was varied, the average gap was 1.48% and 0.29% for the scheme with an incremental and an all-unit discount, respectively.

All the parameters and variables in this study were normalized. The following introduces how to convert the general model to the normalized model. The goal is to find the relationship between the raw parameters and variables and between the normalized parameters and variables. The linear demand model used in this study originated from studies of Singh and Vives (1984) and Dong, Zheng, and Lee (2018), which suggested the following relationships between the price $\bar{p}_i$ and the

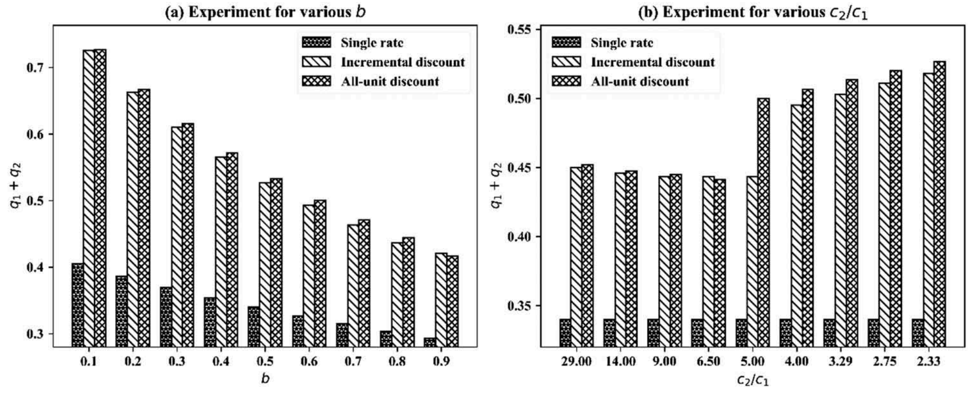

Figure 9. Comparison of the total throughput ($q_1 + q_2$).

quantity $\bar{q}_i$: $\bar{p}_1 = \alpha_1 - \beta\bar{q}_1 - \gamma\bar{q}_2$, $\bar{p}_2 = \alpha_2 - \beta\bar{q}_2 - \gamma\bar{q}_1$, where β and γ ($\beta \geq \gamma > 0$) indicate the substitution parameters between the two container-terminal operators. The unit of α_1 and α_2 is US \$ and the unit of β and γ is US \$ per TEU. The unit of $\bar{p}_1$ and $\bar{q}_1$ is US \$ and TEU, respectively, because container-terminal operators 1 and 2 are assumed to be located in the same port, $\alpha_1 = \alpha_2 = \alpha$ (Dong, Huang, and Ng 2016). The above equations can be rewritten as follows: $\bar{p}_1 = a - \beta\bar{q}_1 - \gamma\bar{q}_2$ and $\bar{p}_2 = a - \beta\bar{q}_2 - \gamma\bar{q}_1$. Let φ be equal to a/β, then the above equations can be rewritten as $\frac{\bar{p}_1}{\beta\varphi} = 1 - \frac{\bar{q}_1}{\varphi} - \frac{\gamma}{\beta}\frac{\bar{q}_2}{\varphi}$ and $\frac{\bar{p}_2}{\beta\varphi} = 1 - \frac{\bar{q}_2}{\varphi} - \frac{\gamma}{\beta}\frac{\bar{q}_1}{\varphi}$. From $q_1 = \frac{\bar{q}_1}{\varphi}, q_2 = \frac{\bar{q}_2}{\varphi}$, $p_1 = \frac{\bar{p}_1}{\beta\varphi}$ and $b = \frac{\gamma}{\beta}$, $p_1 = 1 - q_1 - bq_2$ and $p_2 = 1 - q_2 - bq_1$.

6. Conclusions

This study proposed two revenue-sharing schemes with an incremental and all-unit quantity discount for the design of a concession contract. The revenue-sharing scheme with a discount is a concession contract provided by a port authority, which gives an opportunity for the container-terminal operators to obtain financial benefit for attracting high container traffic volumes. In the revenue-sharing scheme with a discount, if the container volumes of a container-terminal operator are over a predefined threshold, the container-terminal operators can receive a discount on the unit fee per container. This study examined the behaviors of the container-terminal operators, and a two-stage game model was defined, in which the port authority determines the parameters of the revenue-sharing scheme with a discount in the first stage and the container-terminal operators then compete with each other to maximize their profit by determining the container volume in the second stage.

Based on an analysis of the behaviors of container-terminal operators, for a given rental fee rate, it was sufficient to enumerate several discrete candidate values of the discount breakpoint to find the optimal discount breakpoint. Using these useful properties of the optimal discount breakpoint, a simplified search algorithm was suggested for each revenue-sharing scheme with a discount.

Numerical experiments were performed to compare the revenue-sharing scheme with a single rate, with an incremental discount, and with an all-unit discount. The numerical experimental results showed that the revenue-sharing scheme with an all-unit discount gives the highest revenue to the port authority in almost all experimental results and the largest container throughput of the port. The increase in the revenue of the port authority was partly at the cost of the decrease in the profits of the container-terminal operator, but the sum of the revenue of the port authority and the profits of the container-terminal operator was increased by applying revenue sharing with a discount compared to the traditional scheme with a single rate.

Future studies should focus on the following aspects: (1) more than two container-terminal operators can be considered simultaneously; (2) a discount schedule with multiple breakpoints can be applied; and (3) cases where the port authority has an objective other than revenue maximization.

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ORCID

Yanjie Zhou http://orcid.org/0000-0003-2222-9140
Kap Hwan Kim http://orcid.org/0000-0002-9001-5825

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Appendix A. Expressions of Q_1 and Q_2

From $\pi_{11}^* = \pi_{12}^*$, it can be derived

$$Q_1 = \frac{2-b^2}{4-b^2}\left(\frac{2}{(1+b)(2-b)} + \frac{2b(c_2+R_1)-(2-b^2)(R_2+R_1+2c_1)}{(1-b^2)(4-b^2)}\right)$$

and from $\pi_{22}^* = \pi_{23}^*$,

$$Q_2 = \frac{2-b^2}{4-b^2}\left(\frac{2}{(1+b)(2-b)} + \frac{2b(R_2+c_1)-(2-b^2)(R_1+R_2+2c_2)}{(4-b^2)(1-b^2)}\right)$$

Appendix B. Proof of Property 2

Suppose that optimal quantities of terminals are q_{11}^* and q_{21}^* at R_1^* in Case 1. The same amount of revenue may be obtained by the port authority by setting $Q = 0$ and $R_2 = R_1^*$ in Case 3.■

Appendix C. Proof of Property 3

For given values of R_1 and R_2, in both Cases 2 and 3, q_1^* and q_2^* do not change as Q changes. In Case 2, $r_1(q_1^*) = R_1Q + R_2(q_1^* - Q)$ increases with increasing Q, while $r_2(q_2^*)$ remains the same. In Case 3, both $r_1(q_1^*)$ and $r_2(q_2^*)$ increase with increasing Q. Thus, in Case 2, the total revenue is maximized at Q = Q_1, while in Case 3, it is maximized at Q = Q_2. Considering Property 3, the conclusion holds.■

Appendix D. Proof of Property 4

Revenue of the port authority is $R_1Q + R_2(q_{13}^* - Q) + R_1Q + R_2(q_{23}^* - Q)$, for a given Q, as the value of R_1 increases, q_{13}^* and q_{23}^* do not change but the revenue increases. The value of R_1 may increase until the profit of terminal 2 becomes zero.■

Appendix E. Proof of Property 5

Suppose that R_1 and R_2 decrease for a given value of Q. As $R_1$and R_2 decrease, q_1^* and q_2^* in Table 1 increase for all cases. Thus, q_1^*+ q_2^* is maximized when R_1 = R_2 = 0.■

Appendix F. Proof of Property 6

Consider the revenue function with an incremental discount, $Z(Q, R_1, R_2)$ in (13), and the revenue function without a discount, $Z(R)$ in (5). Then, $\max_{Q,R_1,R_2} Z(Q, R_1, R_2) \geq \max_{R_1} Z(Q, R_1, R_2 | Q = \infty, R_2 = 0) = \max_{R} Z(R)$.■

Appendix G. Proof of Property 7

When the value of Q is very large, the container-terminal operators will be in Case 1. As the value of Q becomes smaller, the situation of container-terminal operators will first become Case 4 or Case 8. Let the value of Q at which the situation of container-terminal operators change from Case 1 to Case 4 and from Case 1 to Case 8 be $Q_1$and Q_5 (let Q_5 be the value of Q satisfying $\pi_{28}(R_1, R_2, q_1', \ Q) = \pi_{21}(R_1, R_1, \ q_{11}, q_{21})$), respectively. It can be proven easily that $Q_1 < Q_5$. Thus, for any value of Q, Cases 7 and 8 cannot occur because container-terminal operator 1 is entitled to utilize R_2 earlier than container-terminal operator 2 when Q decreases from a large value. Case 10 cannot occur because Q passes $q_{1(2)}$ earlier than $q_{2(2)}$ when Q decreases from a large value. Thus, Cases 7, 8, and 9 will be removed from further analysis.■

Appendix H. Expressions of Q_1, Q_2, Q_3 and Q_4

$$Q_1 = \frac{\left(1 - c_1 - R_2 - \frac{b - bc_2 - bR_1}{2}\right) + \sqrt{\left(1 - c_1 - R_2 - \frac{b - bc_2 - bR_1}{2}\right)^2 - 2(2 - b^2)(1 - b^2)\left(\frac{1 - R_1}{(2-b)(1+b)} + \frac{bc_2 + b^2 c_1 - 2c_1}{(4-b^2)(1-b^2)}\right)^2}}{(2 - b^2)}$$

$$Q_2 = \frac{1}{(2-b)(1+b)} + \frac{b(c_2 + R_1) + (b^2 - 2)(c_1 + R_2)}{(4 - b^2)(1 - b^2)}$$

$$Q_3 = \frac{2(1 - c_2 - R_2) + b(1 - c_2 - R_1) + \sqrt{(2(1 - c_2 - R_2) + b(1 - c_2 - R_1))^2 - (b^2 + 4b + 4)(1 - c_2 - R_1)^2}}{(b^2 + 4b + 4)}$$

$$Q_4 = \frac{\left(1 - c_2 - R_2 - \frac{b - bc_1 - bR_2}{2}\right) + \sqrt{\left(1 - c_2 - R_2 - \frac{b - bc_1 - bR_2}{2}\right)^2 - 2(2 - b^2)(1 - b^2)\left(\frac{1}{(2-b)(1+b)} + \frac{b(c_1 + R_2) + (b^2 - 2)(c_2 + R_1)}{(4-b^2)(1-b^2)}\right)^2}}{(2 - b^2)}$$

Appendix I. Proof of Property 8

Suppose that optimal annual throughput of container-terminal operators is q_{11}^* and q_{21}^* at R_1^* in Case 1. Then, the same amount of revenue may be obtained by the port authority by setting $q_{11}^* \le Q \le q_{21}^*$ and $R_1 = R_2 = R_1^*$ in Case 3. A similar statement may be made between Cases 2 and 3. Thus, Cases 1 and 2 are dominated by Case 3. Suppose that optimal quantities of terminals are q_{13}^* and q_{23}^* at R_1^* and R_2^* in Case 3. Then, the same amount of revenue may be obtained by the port authority by setting $q_{13}' = q_{13}^*, q_{23}' = q_{13}^*, R_1 = R_1^*, R_2 = R_2^*$, and $Q = q_{13}^*$ in Case 4. A similar statement may be made between Cases 3 and 5. Thus, the optimal solution of Case 3 is dominated by those of Cases 4 and 5.■

Appendix J. Proof of Property 9

Suppose that R_1 and R_2 are given. Table A1 shows how the revenue of the port authority changes as the value of Q changes, which provides candidates of the optimal value of Q.

The revenue function for Case 4 is $\left(\frac{R_2 - bR_1}{2}\right)Q + R_1\left(\frac{1 - c_2 - R_1}{2}\right)$, where $R_1\left(\frac{1 - c_2 - R_1}{2}\right)$ is a fixed component. When $R_2 \ge bR_1/2$, the revenue value for the port authority is increasing with increasing Q. Otherwise, the revenue value for port authority is increasing with decreasing Q. The revenue function for Cases 5 and 6 are $2QR_2$ and $\left(\frac{1 - c_1 - R_2}{2}\right)R_2 + \left(\frac{2R_2 - bR_2}{2}\right)Q$, respectively. Owing to $2R_2 - bR_2 \ge 0$ and $2R_2 \ge 0$, the revenue for the port authority is increasing with the value of Q increasing for Cases 5 and 6. Because the optimal solution may be found only in Cases 4, 5, and 6 from Property 8, it is sufficient to compare the total revenue at boundary values of Q in Table A1. Thus, the conclusion holds.■

Table A1. Change in the revenue of the port authority for each case with a change in Q.

At	Revenue increases as Q	Until it changes to	At boundary value of Q	If
Case 4	Increase	Case 1	Q_1	$R_2 \ge bR_1/2$
Case 4	Decrease	Case 3	Q_2	$R_2 < bR_1/2$
Case 4	Decrease	Case 6	Q_3	$R_2 < bR_1/2$
Case 6	Increase	Case 4	Q_3	-
Case 5	Increase	Case 3	Q_4	-

Appendix K. Proof of Property 10

In Case 1, the total throughput is expressed as $q_1^* + q_2^* = \frac{1-2R_1}{(1+b)(2-b)} + \frac{bc_2-c_1(2-b^2)+bc_1-c_2(2-b^2)}{(1-b^2)(4-b^2)}$. In Case 2, it is $\frac{1-2R_2}{(1+b)(2-b)} + \frac{bc_2-c_1(2-b^2)+bc_1-c_2(2-b^2)}{(1-b^2)(4-b^2)}$. In Case 3, it is $\frac{2}{(2-\mathrm{b})(1+b)} + \frac{\left(\mathrm{b}^2+b-2\right)(R_1+R_2)+\left(\mathrm{b}^2+b-2\right)(c_2+c_1)}{\left(4-\mathrm{b}^2\right)(1-b^2)}$. In Cases 4, 5 and 6, they are $\frac{1+(2-b)Q-c_2-R_1}{2}$, $\frac{1+(2-b)Q-c_1-R_2}{2}$ and $2Q$, respectively. For a given Q, when $R_1$and R_2 decrease, the total throughput does not decrease for all the Cases. Thus, $R_1^* = R_2^* = 0$.∎

Supplemental material for "Optimal concession contract between a port authority and container-terminal operators by revenue-sharing schemes with quantity discount"

Yanjie Zhou and Kap Hwan Kim*
Department of Industrial Engineering, Pusan National University, Busan, Korea
*Corresponding author: Kap Hwan Kim

The following is the supplemental material for the "Optimal concession contract between a port authority and container-terminal operators by revenue-sharing schemes with quantity discount". The supplemental material includes (1) differential profit function of terminal operators; (2) proof of property 1; (3) the Cournot competition model, which was used for the comparison with the model in the paper.

Supplementary document A: Expressions of $\frac{\partial \pi_1}{\partial p_1}$ and $\frac{\partial \pi_2}{\partial p_2}$.

$$\frac{\partial \pi_1}{\partial p_1} = \begin{cases} \frac{1}{1+b} + \frac{bp_2}{1-b^2} + \frac{-2p_1}{1-b^2} + \frac{c_1+R_1}{1-b^2} & q_1 < Q \\ \frac{1}{1+b} + \frac{bp_2}{1-b^2} + \frac{-2p_1}{1-b^2} + \frac{c_1}{1-b^2} + \frac{R_2}{1-b^2} & q_1 \geq Q \end{cases}$$

$$\frac{\partial \pi_2}{\partial p_2} = \begin{cases} \frac{1}{1+b} + \frac{bp_1}{1-b^2} + \frac{-2p_2}{1-b^2} + \frac{c_2+R_1}{1-b^2} & q_1 < Q \\ \frac{1}{1+b} + \frac{bp_1}{1-b^2} + \frac{-2p_2}{1-b^2} + \frac{c_2}{1-b^2} + \frac{R_2}{1-b^2} & q_1 \geq Q \end{cases}.$$

Supplementary document B: Proof of property 1

$$q_{11}^* - q_{21}^* = \frac{bc_2 - c_1(2-b^2) - bc_1 + c_2(2-b^2)}{(1-b^2)(4-b^2)}$$

$= \frac{b(c_2-c_1)+(c_2-c_1)\left(2-b^2\right)}{(1-b^2)(4-b^2)}$

$= \frac{(c_2-c_1)\left(2+b-b^2\right)}{(1-b^2)(4-b^2)}$

Due to $c_2 > c_1$ and $0 < b < 1$, hence $\frac{(c_2-c_1)\left(2+b-b^2\right)}{(1-b^2)(4-b^2)} > 0$. Thus $q_{11}^* > q_{21}^*$ holds.

$q_{21}^* - q_{22}^* = \frac{-R_1}{(1-b^2)(4-b^2)} + \frac{bc_1-c_2(2-b^2)}{(1-b^2)(4-b^2)} - \frac{(b^2-2)(c_2+R_1)+b(c_1+R_2)}{(4-b^2)(1-b^2)}$

$= \frac{-R_1(1-b)(2+b)}{(1-b^2)(4-b^2)} + \frac{bc_1-c_2\left(2-b^2\right)+c_2\left(2-b^2\right)-R_1\left(b^2-2\right)-bc_1-bR_2}{(1-b^2)(4-b^2)}$

$= \frac{-R_1(1-b)(2+b)}{(1-b^2)(4-b^2)} + \frac{-R_1\left(b^2-2\right)-bR_2}{(1-b^2)(4-b^2)}$

$= \frac{-R_1(2-b-b^2)}{(1-b^2)(4-b^2)} + \frac{-R_1\left(b^2-2\right)-bR_2}{(1-b^2)(4-b^2)}$

$= \frac{b(R_1-R_2)}{(1-b^2)(4-b^2)}$

Due to $R_1 > R_2$, hence $\frac{b(R_1-R_2)}{(1-b^2)(4-b^2)} > 0$. Then we can conclude that $q_{21}^* > q_{22}^*$.

Finally, we can conclude that $q_{11}^* > q_{21}^* > q_{22}^*$.

$q_{12}^* - q_{13}^* = \frac{R_2}{(1+b)(2-\mathrm{b})} + \frac{\left(b^2-2\right)(c_1+R_2)+b(c_2+R_1)-bc_2+c_1\left(2-b^2\right)}{(4-b^2)(1-b^2)}$

$= \frac{R_2}{(1+b)(2-\mathrm{b})} + \frac{-\left(2-b^2\right)R_2+bR_1}{(4-b^2)(1-b^2)}$

$= \frac{R_2(1-b)(2+b)}{(4-b^2)(1-b^2)} + \frac{-\left(2-b^2\right)R_2+bR_1}{(4-b^2)(1-b^2)}$

$= \frac{R_2(2-b-b^2)}{(4-b^2)(1-b^2)} + \frac{-\left(2-b^2\right)R_2+bR_1}{(4-b^2)(1-b^2)}$

$= \frac{b(R_1-R_2)}{(4-b^2)(1-b^2)}$

Due to $R_1 > R_2$, $\frac{b(R_1-R_2)}{(4-b^2)(1-b^2)} > 0$ and thus we conclude $q_{12}^* > q_{13}^*$.

$q_{13}^* - q_{23}^* = \frac{bc_2-c_1\left(2-b^2\right)-bc_1+c_2(2-b^2)}{(1-b^2)(4-b^2)}$

$= \frac{b(c_2-c_1)+(c_2-c_1)(2-b^2)}{(1-b^2)(4-b^2)}$

$= \frac{(c_2-c_1)(2+b-b^2)}{(1-b^2)(4-b^2)}$

$= \frac{(c_2-c_1)(2-b)(1+b)}{(1-b^2)(4-b^2)}$

$= \frac{(c_2-c_1)}{(1-b)(2+b)}$

Due to , $\frac{(c_2-c_1)}{(1-b)(2+b)} > 0$ and $q_{13}^* > q_{23}^*$. Thus, $q_{23}^* < q_{13}^* < q_{12}^*$ holds.

From Table 1, we can conclude that: $q_{22}^* < q_{21}^* < q_{11}^* < Q < q_{23}^* < q_{13}^* < q_{12}^*$. Thus, the property 1 holds. ■

Supplementary document C: Cournot competition model

The following context are the Cournot competition model, in which the annual container throughput of terminal operator is considered as decision variable to compete with each other. The Cournot competition model has the same properties compare with the Bertrand model. Due to the properties and proofs of the properties for the Cournot competition model are similar with the Bertrand model, we omitted the properties and proofs of the properties for the Cournot competition model.

1 Revenue-sharing scheme with a single rate

Chen and Liu (2014) proposed a game model for two competitive container-terminal operators with the relationship between the amount of cargo and the terminal handling charge as follows:

$$p_1 = 1 - q_1 - bq_2 \quad \text{(S-1)}$$

$$p_2 = 1 - q_2 - bq_1. \quad \text{(S-2)}$$

The profit of each container-terminal operator then becomes $\pi_i(R) = p_i q_i - (c_i + R)q_i$. Chen and Liu (2014) obtained the Nash equilibrium (NE) analytically for the competitive game as follows:

$$q_1^* = \frac{1-R}{2+b} + \frac{bc_2-2c_1}{4-b^2} \text{ and } q_2^* = \frac{1-R}{2+b} + \frac{bc_1-2c_2}{4-b^2}\,. \tag{S-3}$$

The profit becomes

$$\pi_i^*(R) = (q_i^*)^2 \text{ for } i = 1, 2. \tag{S-4}$$

Note that $q_1^* > q_2^* \geq 0$ and $\pi_1^* > \pi_2^* \geq 0$.

(S-5)

From $q_2^* \geq 0$,

$$R \leq \overline{r} \equiv \frac{2(1-c_2)-b(1-c_1)}{2-b}\,, \tag{S-6}$$

and from $\overline{r} \geq 0$,

$$c_2 < \bar{c}_2 \equiv \frac{2-b+bc_1}{2}\,. \tag{S-7}$$

The revenue of the port authority, $Z(R)$, becomes $R(q_1 + q_2)$. The optimal rental fee per TEU may be derived (Chen and Liu, 2014) as follows:

$$R^* = \frac{1}{2} - \frac{c_1+c_2}{4}. \tag{S-8}$$

From $R^* < \overline{r}$, it can be shown that $c_2 \leq \hat{c}_2 \equiv \frac{3bc_1+4-2b}{6+b}\,(< \bar{c}_2)$, which is a tighter upper bound than $\hat{c}_2 \equiv \frac{4+3bc_1+2c_1-2b}{6+b}$ suggested by Chen and Liu (2014).

2 Revenue-sharing scheme with a quantity discount

2.1 Revenue-sharing scheme with an incremental discount

2.1.1 Optimal behaviours of terminal operators

The profit function for container-terminal operator i may be expressed as follows:

$$\pi_i = \begin{cases} p_i q_i - (c_i + R_1) q_i & q_i < Q \\ p_i q_i - c_i q_i - R_1 Q - R_2 (q_i - Q) & q_i \geq Q. \end{cases} \tag{S-11}$$

Given (R_1, R_2, Q), which is provided by the port authority, the container-terminal operators compete with each other to decide their own optimal cargo amounts (q_1^*, q_2^*) by solving the following problems:

$$\max_{q_1 \geq 0} \pi_1 = \begin{cases} p_1 q_1 - (c_1 + R_1) q_1 & q_1 < Q \\ p_1 q_1 - c_1 q_1 - R_1 Q - R_2 (q_1 - Q) & q_1 \geq Q \end{cases} \tag{S-12}$$

$$\max_{q_2 \geq 0} \pi_2 = \begin{cases} p_2 q_2 - (c_2 + R_1) q_2 & q_2 < Q \\ p_2 q_2 - c_2 q_2 - R_1 Q - R_2 (q_2 - Q) & q_2 \geq Q. \end{cases} \tag{S-13}$$

From the first order necessary conditions for maximizing π_i ($\frac{\partial \pi_1}{\partial q_1} = \frac{\partial \pi_2}{\partial q_2} = 0$),

$$\frac{\partial \pi_1}{\partial q_1} = \begin{cases} 1 - 2q_1 - b q_2 - c_1 - R_1 & q_1 < Q \\ 1 - 2q_1 - b q_2 - c_1 - R_2 & q_1 \geq Q \end{cases} \tag{S-14}$$

$$\frac{\partial \pi_2}{\partial q_2} = \begin{cases} 1 - 2q_2 - b q_1 - c_2 - R_1 & q_2 < Q \\ 1 - 2q_2 - b q_1 - c_2 - R_2 & q_2 \geq Q. \end{cases} \tag{S-15}$$

Hence, $\frac{\partial^2 \pi_i}{\partial (q_i)^2} = -2$ and $\frac{\partial^2 \pi_i}{\partial q_1 q_2} = -b$. Thus, the Hessian matrix, H>0, implies that π_i is convex with respect to q_1 and q_2. If the equations $\frac{\partial \pi_1}{\partial q_1} = 0$ and $\frac{\partial \pi_2}{\partial q_2} = 0$ are solved simultaneously, the NE solution of the cargo amounts (q_1^*, q_2^*) can be obtained.

Considering whether two terminal operators are entitled to a concession discount or not, four different cases are shown as follows:

Case 1: Both container-terminal operators 1 and 2 are not entitled to the discount.

Case 2 (primary): Container-terminal operator 1 is not entitled to the discount but container-terminal operator 2 is.

Case 3: Container-terminal operator 2 is not entitled to the discount but container-terminal operator 1 is.

Case 4: Both container-terminal operators 1 and 2 are entitled to the discount.

By solving the first order necessary conditions for cases 1, 2, 3, and 4, the optimal cargo amount (q_1^*, q_2^*) and optimal profit (π_1^*, π_2^*) can be obtained, as shown in Supplementary Table 1.

Supplementary Table 1. Optimal cargo amounts (q_1^*, q_2^*) and optimal profit (π_1^*, π_2^*) for container-terminal operators 1 and 2 for revenue-sharing scheme with an incremental discount.

Cases	q_1^*, q_2^*	π_1^*, π_2^*
1	$q_1^*(R_1, R_1, q_1, q_{21}^*) = q_{11}^* = \frac{bc_2 - 2c_1 + 2 - b - 2R_1 + bR_1}{4 - b^2}$ $q_2^*(R_1, R_1, q_{11}^*, q_2) = q_{21}^* = \frac{bc_1 - 2c_2 + 2 - b - 2r_1 + br_1}{4 - b^2}$	$\pi_1^*(R_1, R_1, q_{11}^*, q_{21}^*) = \pi_{11}^* = (q_{11}^*)^2$ $\pi_2^*(R_1, R_1, q_{11}^*, q_{21}^*) = \pi_{21}^* = (q_{21}^*)^2$

2	$q_1^*(R_2, R_1, q_1, q_{22}^*) = q_{12}^* = \dfrac{bc_2 - 2c_1 + 2 - b - 2R_2 + bR_1}{4 - b^2}$ $q_2^*(R_2, R_1, q_{12}^*, q_2) = q_{22}^* = \dfrac{bc_1 - 2c_2 + 2 - b - 2R_1 + bR_2}{4 - b^2}$	$\pi_1^*(R_2, R_1, q_{12}^*, q_{22}^*) = \pi_{12}^* = (q_{12}^*)^2 + R_2Q - R_1Q$ $\pi_2^*(R_2, R_1, q_{12}^*, q_{22}^*) = \pi_{22}^* = (q_{22}^*)^2$
3	$q_1^*(R_2, R_2, q_1, q_{23}^*) = q_{13}^* = \dfrac{bc_2 - 2c_1 + 2 - b - 2R_2 + bR_2}{4 - b^2}$ $q_2^*(R_2, R_2, q_{13}^*, q_2) = q_{23}^* = \dfrac{bc_1 - 2c_2 + 2 - b - 2R_2 + bR_2}{4 - b^2}$	$\pi_1^*(R_2, R_2, q_{13}^*, q_{23}^*) = \pi_{13}^* = (q_{13}^*)^2 + R_2Q - R_1Q$ $\pi_2^*(R_2, R_2, q_{13}^*, q_{23}^*) = \pi_{23}^* = (q_{23}^*)^2 + R_2Q - R_1Q$

2.1.2 Optimal concession contracts

Suppose that (R_1, R_2) are given. When Q is very large, both terminal operators do not want to accept the discount rate, which is Case 1. As Q becomes smaller, there are two possible routes for the transition of the cases: Case 1 → Case 2 → Case 4 or Case 1 → Case 3 → Case 4. Table 2 lists the boundary values of Q, which changes the situation from one case to another. For example, suppose that the current situation corresponds to case 1, which means Q is too large for both terminals to utilize the discounted unit rate. When Q decreases, π_{13}^* increases and then exceeds π_{11}^* at a specific boundary value of Q. The boundary value of Q, which is denoted as Q_{11}, may be obtained by solving $\pi_{11}^* = \pi_{13}^*$. In the same way, the other boundary values may be obtained, as shown in Supplementary Table 2.

Supplementary Table 2 . Boundary values of Q between cases.

From	To	Boundary value of Q

Case 1	Case 2	From $\pi^*_{11} = \pi^*_{13}$, $Q = Q_1 \equiv \frac{4(bc_2-2c_1+2-b-R_1-R_2+bR_1)}{(4-b^2)^2}$
Case 2	Case 3	From $\pi^*_{23} = \pi^*_{24}$, $Q = Q_2 \equiv \frac{4(bc_1-2c_2+2-b-R_1-R_2+bR_2)}{(4-b^2)^2}$

The properties 1, 2, and 3 are hold for the revenue-sharing scheme with an incremental discount by using Cournot competition model. Due to the method to proof for these properties are similar, we omit the proofs for these properties.

2.2 Revenue-sharing scheme with an all-unit discount

2.2.1 Optimal behaviours of terminal operators

The NE quantity may be derived easily using the first order necessary conditions, as listed in Table 3.

Supplementary Table 3. Optimal cargo amounts (q_1^*, q_2^*) and optimal profit (π_1^*, π_2^*) for container-terminal operator 1 and 2 for a revenue-sharing scheme with all-unit discount.

Case	Simplified notation	Formal notation	Expression
1	q_{11}	$q_1^*(R_1, R_1, q_1, q_{21})$	$\frac{bc_2-2c_1+2-b-2R_1+bR_1}{4-b^2}$
	q_{21}	$q_2^*(R_1, R_1, q_{11}, q_2)$	$\frac{bc_1-2c_2+2-b-2R_1+bR_1}{4-b^2}$
2	q_{12}	$q_1^*(R_2, R_2, q_1, q_{22})$	$\frac{bc_2-2c_1+2-b-2R_2+bR_2}{4-b^2}$
	q_{22}	$q_2^*(R_2, R_2, q_{12}, q_2)$	$\frac{bc_1-2c_2+2-b-2R_2+bR_2}{4-b^2}$
3	q_{13}	$q_1^*(R_2, R_1, q_1, q_{23})$	$\frac{bc_2-2c_1+2-b-2R_2+bR_1}{4-b^2}$
	q_{23}	$q_2^*(R_2, R_1, q_{13}, q_2)$	$\frac{bc_1-2c_2+2-b-2R_1+bR_2}{4-b^2}$
4	q'_{13}	$q_1^*(R_2, R_1, Q, q'_{23})$	Q

	q'_{23}	$q_2^*(R_2, R_1, Q, q_2)$	$\frac{1-bQ-c_2-R_1}{2}$
5	q'_{12}	$q_1^*(R_2, R_2, q_1, Q)$	$\frac{1-bQ-c_1-R_2}{2}$
	q'_{22}	$q_2^*(R_2, R_2, q'_{12}, Q)$	Q
6	q''_{12}	$q_1^*(R_2, R_2, Q, Q)$	Q
	q''_{22}	$q_2^*(R_2, R_2, Q, Q)$	Q

From $c_2 > c_1$, $R_2 < R_1$, and $0 < b < 1$, the following inequalities follow: $q_{11} < q_{12} < q_{13}$, $q_{23} < q_{21} < q_{22}$, $q_{11} > q_{21}$, $q_{12} > q_{22}$.

4.2.2 Optimal concession contracts

Supplementary Table 4. Change of the revenue of the port authority of each case for change of Q

At	Revenue increases as Q	Until it changes to	At boundary value of Q	If
Case 4	Increase	Case 1	Q_1	$R_2 \geq bR_1/2$
Case 4	Decrease	Case 3	Q_2	$R_2 < bR_1/2$
Case 6	Increase	Case 4	Q_3	-
Case 5	Increase	Case 3	Q_4	-

- Denotes that for given any R_1 and R_2

The details for deriving Q_i is presented as follows. Q_1 is the value of Q satisfying that $\pi_{13}(R_2, R_1, Q, q'_{23}) = \pi_{11}(R_1, R_1, q_{11}, q_{21})$ and Q_2 is the value of q_{13}. Q_3 is the value of Q satisfying that $\pi_{22}(R_2, R_2, Q, Q) = \pi_{23}(R_2, R_1, Q, q'_{23})$. Q_4 is the value of Q satisfying that $\pi_{22}(R_2, R_2, q_{12}, Q) = \pi_{23}(R_2, R_1, q_{13}, q_{23})$. By solving the above equations of Q, we can derive the Q_1, Q_2, Q_3, and Q_4 which are presented as follows:

$$Q_1 = \frac{bc_2-2c_1+2-b-2R_2+bR_1}{4-2b^2} - \frac{\sqrt[2]{\left(\frac{bc_2-2c_1+2-b-2R_2+bR_1}{2}\right)^2+2(b^2-2)\left(\frac{bc_2-2c_1+2-b-2R_1+bR_1}{4-b^2}\right)^2}}{b^2-2}$$

$$Q_2 = \frac{bc_2-2c_1+2-b-2R_2+bR_1}{4-b^2}$$

$$Q_3 = \frac{(b-bc_2-bR_1-2c_2-2R_2+2)}{(b^2+4b+4)} + \frac{\sqrt{(b-bc_2-bR_1-2c_2-2R_2+2)^2-\left((b+2)(1-c_2-R_1)\right)^2}}{(b^2+4b+4)},$$

$$Q_4 = \frac{bc_1-2c_2+2-b-2R_2+bR_2}{4-2b^2} - \frac{\sqrt[2]{\left(\frac{bc_1-2c_2+2-b-2R_2+bR_2}{2}\right)^2+2(b^2-2)\left(\frac{bc_1-2c_2+2-b-2R_1+bR_2}{4-b^2}\right)^2}}{b^2-2}.$$

The properties 7, 8, and 9 hold for the revenue-sharing scheme with all unit discount by using Cournot competition model. Due to the method to prove these properties are similar, we omit the proofs for these properties.

Reference

Yanjie Zhou and Kap Hwan Kim, Optimal concession contract between a port authority and container-terminal operators by revenue-sharing schemes with quantity discount. *Maritime Policy & Management*, https://doi.org/10.1080/03088839.2019.1707314