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Optimal parameters in concession contracts between container terminal operators and investors

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ABSTRACT
International investments in the port development become popular including the cases of Chinese companies who are continuously signing concession contracts with many other countries to promote the Belt and Road Initiative. One important issue in a concession contract between an investor and a container terminal operator is how the rental fee is calculated. This study discusses how the port investor optimises the parameters of fixed and variable rental fees for both the cases with a deterministic and an uncertain cargo demand. This study analyzes cases with an uncertain cargo demand in which terminal operators and the port investor may have the same or different degrees of uncertainty on cargo demand. The uncertainty in the cargo demand was found to decrease the revenue of the port investor and increase the profits of the terminal operators.

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Belt and Road Initiative; concession contract; fixed and variable rental fee; container terminal operator; port investor

1. Introduction

Global terminal operators (GTOs) have been investing in more and more oversea container terminals. Since the Chinese government proposed the Belt and Road policy in 2013, Chinese companies are also continuously signing concession contracts with many other countries to construct or rent container terminals along the Maritime Silk Road. According to Wang et al. (2019), the number of container terminals that three major Chinese terminal operators (Hutchison Ports, COSCO Shipping Ports, and China Merchants Ports) have invested in along the Maritime Silk Road between 2005 and 2017 has increased from 13 to 51. Table 1 classifies the cases of investments by two Chinese companies according to the types of investment (Huo et al. 2019). Five investment models have been found, including joint venture and co-operation, joint venture, acquisition, BOT (build operate transfer), concession, and mergers and acquisitions. There have been other studies related to the overseas investment based on the Belt and Road Initiative (Chen et al. 2019; Ruan et al. 2019).

After the investment, a GTO may play the role of a investor as well as an operator of the container terminals. As an operator, the GTO will compete with other terminal operators in the same port. As a landlord, however, it will design a concession contract for terminal operators of the constructed terminals. Therefore, this study discusses how to compete with other terminal operators as a terminal operator and how to design concession contracts between container terminal operators and an investor. The contents in this study may be applied not only to GTOs investing oversea port development but also to port authorities or other government agencies developing domestic or oversea ports.

Specifying a concession contract between an investor and container terminal operator is a complicated task (Notteboom, Pallis, and Farrell 2012), which involves determining the rental fee schemes, duration of the concession contract (Theys and Notteboom 2010), risk sharing (Cruz and Marques

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Table 1. Classification of the investment cases according to the type of investment.

Investment companies of China	Investment type				
	Joint venture and cooperation	Joint venture, acquisition	BOT (build operate transfer)	Concession	Mergers and acquisitions
COSCO Shipping Ports	40%	0%	0%	10%	50%
China Merchants Ports	33%	11%	11%	11%	33%

2012), etc. Concession contracts between a port authority and container terminals have been addressed widely in previous studies. Parola, Tei, and Ferrari (2012) examined how to manage concession contracts in Italy. De Langen, Van Den Berg, and Willeumier (2012) proposed a new award process to grant concessions to a large container terminal in the port of Rotterdam. Suárez-Alemán, Serebrisky, and Ponce De León (2018) studied the port competition in Latin America and the Caribbean, analyzed the concession processes in the area, and emphasized the importance of a concession contract. Van Niekerk (2005) studied the port concession process in developing countries.

This study investigated how to optimise the parameters of fixed and variable rental fees in concession contracts between the port investor and container terminal operators. Several types of concession contracts are available: a lump sum, in which a fixed amount of rental fee is paid at the beginning; the annual rent, in which a fixed amount of rental fee is paid every year; unit fee revenue sharing, in which the annual rental fee is proportional to the throughput; and mixed policies, in which an annual rental fee and unit fee are used together, which is the main issue in this study.

Chen and Liu (2014) proposed a two-stage game model to investigate concession contracts for landlord port authorities to maximise the total revenue fee. In the proposed two-stage game, there are one port authority and two competitive container terminal operators, which aim to maximise their profits. In the first stage of the game, the port authority provides a set of parameters of the concession contract to maximise the total revenue, which is collected from two container terminal operators. In the second stage of the game, the two container terminal operators compete with each to maximise their profit by deciding the container throughput. Chen and Liu (2015) used the same two-stage game model reported by Chen and Liu (2014) but to maximise the traffic volume of the entire port instead of the total revenue of the port authority. Chen, Lin, and Liu (2017) extended Chen and Liu's (2014) study by assuming that the container terminal operators compete with each other using the terminal handling charge, instead of using the cargo amount. Liu et al. (2018) extended Chen and Liu's study (2015) by assuming that each terminal has a constraint on the minimal throughput requirement. Han, Chen, and Liu (2018) also extended Chen and Liu's study (2014) with different pursuing objectives, including the weighted sum of revenues and throughput benefits as well as social welfare. Lirn, Thanopoulou, and Beresford (2003) introduced a survey result showing that loading/discharging cost is one of the three most important considerations when a vessel carrier selects a transshipment container terminal. Dandotiya et al. (2011) studied the optimal pricing and terminal location for a rail–truck intermodal service. Ishii et al. (2013) proposed a non-cooperative game theoretic model, where each port selects the port charges strategically in the timing of the port capacity investment and applies the model to the case of competition between Busan and Kobe. Munim, Saeed, and Larsen (2019) analyzed the gains by transforming the port governance from the tool port governance model to the landlord port governance model using the competitive and cooperative game theories. Recently, Zhou and Kim (2019) proposed revenue-sharing schemes with a quantity discount to optimise the concession contract between a port authority and container-terminal operators.

The game in this study is a two-stage model, in which the port investor proposes a fee scheme to maximise their own revenue in the first stage, while, given the fee scheme, terminal operators 1 and 2 choose the cargo amounts or handling charges simultaneously and independently in the second stage to maximise their profits. The perfect Nash equilibrium may be obtained by the backward induction.

The contributions of this study are summarised as follows. This study addressed the design problem of the concession contract for the port investor under the assumption that the cargo demand is deterministic and uncertain. In the case with a deterministic cargo demand, this study derived new expressions for the optimal parameters of a mixed concession scheme, which was studied by Chen and Liu (2014) and Chen, Lin, and Liu (2017). The approach of this study is different from those reported by Chen and Liu (2014) and Chen, Lin, and Liu (2017) in that this study attempted to optimise the parameters of the fixed annual fee and unit fee simultaneously, which must provide a better solution than that from the sequential decision, and a correct constraint on the profit function of terminal operators was used. Detailed explanations about the differences between the two approaches are provided in section 5.3.1. Numerical experiments showed that the proposed expressions give better performance than those reported by Chen and Liu (2014) and Chen, Lin, and Liu (2017) in terms of the revenue collected by the port investor. For the case with an uncertain cargo demand, this study found that the optimal solution depends heavily on the maximum risk of loss of the terminal operators, which was introduced in this study and means the maximum probability that the profit of the terminal operators becomes negative. The model with the demand uncertainty in Liu et al. (2018) is a special case of the problem in this study, in which the maximum risk of loss is 0.5 (refer to Property 4.2). Three different cases with an uncertain demand were analyzed: the case where the cargo demand is uncertain to all the participants in the same degree; the case where the cargo amount of the port is uncertain to the port investor when the parameters for the rental fee are determined but are known to container terminal operators; and the case where the cargo demand is uncertain to all the participants but the the uncertainty is larger to the port investor than to terminal operators. Note that this study proposed these models for the first time. The optimal expressions or a solution procedure for the optimal solution are provided for the three cases with an uncertain cargo demand. The cases with a deterministic demand and those with an uncertain demand are compared with each other, and any interesting insights are discussed.

The remainder of the paper is organized as follows. Section 2 derives the optimal expressions for the fixed and variable rental fees in concession contracts with a quantity competition model. Section 3 addresses the concession contracts with the price competition model. Section 4 introduces the cargo demand uncertainty. Section 5 presents the results of sensitivity analyses, compares the results of this study with those reported elsewhere, and compares various cases proposed in this study. The conclusions are given in the final section.

2. Fixed and variable rental fees in the quantity competition model with a cargo demand known to all participants (case QD)

This section derives the optimal expressions of the parameters for the fixed and variable rental fee scheme. Section 2.1 describes how the terminal operators behave for a given rental fee scheme. Section 2.2 discusses how the port investor can determine the parameters of the fixed and variable rental fee considering the behaviors of the terminal operators.

The following assumptions are introduced for the formulation:

1) Two container terminal operators compete with each other in a port, and the two container terminal operators provide the same quality of services to the customers. This assumption simplifies the analysis and has been introduced in previous studies (Chen and Liu 2014; Chen and Liu 2015; Chen, Lin and Liu 2017; Liu et al. 2018; Zhou and Kim 2019).
2) The same rental fee scheme with a fixed and unit rental fee is provided to two container terminal operators. It is reasonable to assume that the same rental fee scheme is proposed to the terminal operators in the same port for fairness.
3) The relationship between the terminal handling charge and the cargo amounts of the container terminal operators is linear. A linear relationship is most popular for representing the demand

function in this field. In addition, this assumption allows the mathematical analysis to provide the optimal solution of the closed-form expressions in many cases. Chen and Liu (2014, 2015), Chen, Lin and Liu (2017), Liu et al. (2018), Dong, Zheng, and Lee (2018), and Zhou and Kim (2019) also adopted this assumption.

2.1. Competition between two container terminal operators

The notations and decision variables used in this study are as follows:

Notations:	
i	Index for the container terminals and $i = 1$ or 2.
p_i	Terminal handling charge per container of container terminal operator i
b	Service substitution degree of the container terminal operators and $b \in (0, 1)$.
c_i	Variable cost rate of container terminal operator i and $c_i \in (0, 1)$. It is assumed that $c_2 > c_1$.
$\pi_i(f, r)$	Profit function of container terminal operator i under the mixed rental fee scheme.
$Z(f, r)$	Revenue function of the port investor under the mixed rental fee scheme.
Decision variables:	
q_i	Cargo amount (TEU) of container terminal operator i.
r	Unit fee that is charged by the port investor.
f	Fixed fee that is charged by the port investor.

Chen and Liu (2014) proposed a game model for two competitive terminal operators with the relationship between the cargo amount and terminal handling charge as follows:

$$p_1 = 1 - q_1 - bq_2 \tag{1}$$

$$p_2 = 1 - q_2 - bq_1 \tag{2}$$

The mixed rental fee scheme is defined by two parameters, including the fixed fee (f), and the unit fee (r). For a given f and r, the annual rental fee may be expressed as $f + rq_i$. Thus, the profit maximising problem of terminal operators may be expressed as

(T-QD)

$$\max_{q_i} \pi_i(f, r) = p_i q_i - (c_i + r)q_i - f \quad \text{for } i = 1, 2, \tag{3}$$

subject to
$q_i \geq 0, \ i = 1, \ 2,$
and constraints (1) and (2).

With fixed values of f and r, the terminal operators will optimise q_i to maximise their profits simultaneously. We denote the optimal value of q_i by q_i^{d*} for distinguishing the optimal solutions in the deterministic case from those with cargo amount uncertainties. By optimising q_i for $= 1, \ 2$, Chen and Liu (2014) obtained the optimal q_1^{d*} and q_2^{d*}, as follows:

$$q_1^{d*} = \frac{1-r}{2+b} + \frac{bc_2 - 2c_1}{4 - b^2}$$
$$\text{and} \tag{4}$$
$$q_2^{d*} = \frac{1-r}{2+b} + \frac{bc_1 - 2c_2}{4 - b^2}.$$

From the assumption that $c_2 > c_1$, it can be derived easily that $q_1^{d*} > q_2^{d*} \geq 0$. By replacing q_1 and q_2 in equation (3) with equation (4), the optimal profit function can be derived as (Chen and Liu 2014)

$$\pi_i^{d*}(f, r) = (q_i^{d*})^2 - f \quad \text{for } i = 1, \ 2. \tag{5}$$

From $q_1^{d*} > q_2^{d*} \geq 0$, $\pi_1^{d*}(f, r) > \pi_2^{d*}(f, r) \geq 0$. From $q_2^{d*} \geq 0$, an upper bound of r, $\bar{r}$, can be found as follows:

$$r \leq \bar{r} \equiv \frac{2(1 - c_2) - b(1 - c_1)}{2 - b}, \tag{6}$$

From the obvious condition, $\bar{r} \geq 0$, an upper bound of c_2, $\bar{c}_2$, is as follows:

$$c_2 < \bar{c}_2 \equiv \frac{2 - b + bc_1}{2}. \tag{7}$$

2.2. Optimising the parameters of the fixed and variable fees by the port investor

The revenue function of the port investor from both container terminal operators can be expressed as

$$Z(f, r) = 2f + r(q_1 + q_2). \tag{8}$$

This study attempted to determine the optimal values of f and r simultaneously.

The problem of maximising the revenue of the port investor is

(P-QD)

$$\max_{f,r} Z = 2f + r(q_1^{d*} + q_2^{d*}), \tag{9}$$

subject to

$$0 \leq r \leq \bar{r},\ \ \pi_1^{d*} \geq 0 \text{ and } \pi_2^{d*} \geq 0. \tag{10}$$

Property 2.1: With quantity competing terminal operators (P-QD), the optimal parameters of the fixed and variable rental fee scheme, (r^{d*}, f^{d*}), becomes

$$r^{d*} = \frac{(4 + b^2)c_2 - (4 + 4b - b^2)c_1 + 4b - 2b^2}{4(1 + b)(2 - b)} \tag{11}$$

and $f^{d*} = (q_2^{d*})^2$. In addition, for the problem (P-QD) to be feasible, it should hold that $c_2 \leq \hat{c}_2 \equiv \dfrac{4 + 3bc_1 + 2c_1 - 2b}{6 + b}$. Note that $\bar{c}_2 > \hat{c}_2$.

Proof) See Appendix 1.

3. Fixed and variable rental fees in the price competition model with a cargo demand known to all (case PD)

In this section, it is assumed that the container terminal operators compete with each other by optimising the terminal handling charge, p_1 and p_2, as decision variables. Both competition models are considered to be realistic because the terminal operators compete using the cargo handling quantity, which is equivalent to the handling capacity, in the stage of terminal development, while they compete with each other using the handling price in the stage of operation. Previous studies attempted to compare the two competition models (Hackner 2000; Hinloopen and Vandekerckhove 2009; Mukherjee 2011; Silva and Verhoef 2013; Chen, Lin, and Liu 2017). Flath (2012) supported both competition models based on a survey of real cases.

3.1. Competition between two container terminal operators

Equations (1) and (2) can be rewritten as

$$q_1 = \frac{1}{1+b} - \frac{p_1}{1-b^2} + \frac{bp_2}{1-b^2} \tag{12}$$

and

$$q_2 = \frac{1}{1+b} - \frac{p_2}{1-b^2} + \frac{bp_1}{1-b^2}. \tag{13}$$

The profit maximisation problem of container terminal operator i then becomes
(T-PD)

$$\max_{p_i} \pi_i(f,\ r) = p_i q_i - (c_i + r)q_i - f, \quad i = 1,\ 2. \tag{14}$$

subject to $p_i \geq 0,\ \ i = 1,\ 2,$
and constraints (12), and (13).
The profit function for container terminal operators 1 and 2 can be rewritten as follows:

$$\pi_1(f,\ r) = (p_1 - c_1 - r)\left(\frac{1}{1+b} - \frac{p_1}{1-b^2} + \frac{bp_2}{1-b^2}\right) - f,$$

$$\pi_2(f,\ r) = (p_2 - c_2 - r)\left(\frac{1}{1+b} - \frac{p_2}{1-b^2} + \frac{bp_1}{1-b^2}\right) - f.$$

With fixed values of f and r, the container terminal operators will optimise p_i to maximise their profits simultaneously. By optimising the profit functions for $i = 1,\ 2$, Chen, Lin, and Liu (2017) obtained the optimal p_1^{d*}, p_2^{d*}, q_1^{d*}, and q_2^{d*} as follows:

$$p_1^{d*} = \frac{1-\mathrm{b}+r}{2-b} + \frac{2c_1 + bc_2}{4-b^2},\ p_2^{d*} = \frac{1-\mathrm{b}+r}{2-b} + \frac{2c_2 + bc_1}{4-b^2}$$

$$q_1^{d*} = \frac{1-r}{(1+b)(2-b)} + \frac{c_2 b - c_1(2-b^2)}{(1-b^2)(4-b^2)},\ \text{and}\quad q_2^{d*} = \frac{1-r}{(1+b)(2-b)} + \frac{c_1 b - c_2(2-b^2)}{(1-b^2)(4-b^2)}.$$

Note that $q_1^{d*} > q_2^{d*}$. From $q_2^{d*} \geq 0$, $\mathrm{r} \leq \bar{r} \equiv \dfrac{(2-b^2)(1-c_2) - \mathrm{b}(1-c_1)}{(1-b)(2+b)}$, and from $\bar{r} \geq 0$, $c_2 < \bar{c}_2 \equiv 1 - \dfrac{b - bc_1}{2-b^2}$. $\pi_i^{d*}(f, r) = (1-b^2)(q_i^{d*})^2 - f$ for $i = 1,\ 2$. (Chen, Lin, and Liu 2017).

3.2. Optimising the parameters of the fixed and variable fees by the port investor

The problem of maximising the revenue of the port investor under the price competition model (P-PD) has the same formulation as (P-QD), and its optimal parameters of the mixed concession contract may be obtained as follows for the price competition model.

Property 3.1: With price-competing container terminal operators, the optimal parameters of the fixed and variable rental fee scheme, (r^{d*}, f^{d*}), becomes $r^{d*} = \dfrac{(4-3b^2)c_2 - (4-b^2+4b)c_1 + 2b(2+b)}{4(2+b)}$ and $f^{d*} = (1-b^2)(q_2^{d*})^2$. In addition, for the problem (P-PD) to be feasible, it should hold that $c_2 \leq \hat{c}_2 = \dfrac{(8-8\mathrm{b}-2b^2+2b^3) + (4+4b-5b^2+b^3)c_1}{12-4b-7b^2+3b^3}$. Note that $\hat{c}_2 < \bar{c}_2$.

Proof) See Appendix 2.

It is meaningful that $\hat{c}_2$ is a tighter upper bound of c_2 than $\bar{c}_2$, that is, $c_2 \leq \hat{c}_2 < \bar{c}_2$.

4. Cases with uncertain cargo demand

In previous sections, it was assumed that the amount of cargo of the port is known in advance. However, the amount of cargo demand may be uncertain when the port investor and terminal operators make decisions. This section addresses three cases with uncertain cargo demand. In the first case, the cargo demand is uncertain to all the participants, the port investor and terminal operators, in the same degree, which we call the case with symmetrically uncertain demand (case U). Considering that the rental contract between the investor and a terminal operator is usually made on a long term basis, it may be realistic to assume that the port investor makes a decision based on the higher uncertainty in the future cargo amount of the port than terminal operators. Thus, as the second case, we assume that the port investor makes a decision based on the high uncertainty in the future cargo amount of the port but, terminal operators may determine the terminal handling charge based on a known the cargo demand. This case is called the case with 'asymmetric (case A)' information on cargo demand in this paper. In the third case, the terminal operators also have uncertain information on the cargo amount but whose uncertainty is smaller than that of the port investor, which is called the case with 'partially asymmetric (case P)' information on cargo demand in this paper.

4.1. Case with symmetric uncertainty on the cargo demand (case U)

In this case, both the port investor and terminal operators have uncertain information on the cargo amount of the port. That is, all the participants in this game have the same degree of uncertainty on the cargo demand. Because the cargo amount is uncertain to the container terminal operators, based on the model in Section 2, the demand-price relationships may be expressed as follows:

$$p_1 = 1 + \alpha - q_1 - bq_2, \tag{15}$$

$$p_2 = 1 + \alpha - q_2 - bq_1, \tag{16}$$

where α represents the uncertainty on the cargo demand and is assumed to follow a normal distribution, $N(0, \sigma_\alpha^2)$.

Then, each terminal operator attempts to maximise the expected value of its own profit as follows:

(T-U)

$$\max_{q_i} E_\alpha[\pi_i(f, r)] = E_\alpha[p_i q_i - (c_i + r)q_i - f] \quad \text{for } i = 1, 2, \tag{17}$$

subject to $q_i \geq 0$, $i = 1, 2$,

and constraints (15) and (16).

The objective function may be rewritten as

$$E_\alpha[\pi_1(f, r)] = E_\alpha[(1 + \alpha - q_1 - bq_2 - c_1 - r)q_1 - f] \tag{18}$$

$$E_\alpha[\pi_2(f, r)] = E_\alpha[(1 + \alpha - q_2 - bq_1 - c_2 - r)q_2 - f] \tag{19}$$

Considering $E_\alpha(\alpha) = 0$ and the above two equations. (T-U) becomes the same as (3). Hence, $q_i^{u*} = q_i^{d*}$ and $E_\alpha[\pi_i^{u*}(f, r)] = (q_i^{d*})^2 - f$ for $i = 1$ and 2. The revenue-maximising problem of the port investor for the case with symmetrically uncertain information (U) is

(P-U)

$$\max_{r,f} E_\alpha(Z) = E_\alpha[2f + r(q_1^{u*} + q_2^{u*})],$$

subject to $P\{\pi_i^{u*}(f, r) \geq 0\} \geq 1 - \delta$ for $i = 1$ and 2, where δ is the maximum allowed probability that a terminal has a loss, which is called the 'maximum risk of loss' in this paper. From the viewpoint of the port investor, he/she attempts to maximise their revenue under the condition that all the terminal operators have a nonnegative profit. On the other hand, the profit non-negativity condition needs to be expressed in a probabilistic term, instead of a deterministic term as in Section 2, considering the uncertainty in the cargo demand as follows: $p[\pi_i^{u*}(f,\ r) \geq 0] \geq 1 - \delta$ for $i = 1$ and 2. It is reasonable to assume that the value of δ is much smaller than 0.5, for example, a value between 0.01 and 0.10.

The following property provides the optimal values of the parameters of the rental fee scheme:

Property 4.1: For the problem of (P-U), the optimal parameters of the fixed and variable rental fee scheme, (f^{u*}, r^{u*}), becomes

$$f^{u*} = z_\delta \sigma_\alpha q_2^{u*} + (q_2^{u*})^2 \tag{20}$$

and

$$r^{u*} = \min\left\{ \overset{=u}{r},\ \frac{1}{4+4b}\left(-2z_\delta\sigma_\alpha(2+b) - 4 - \frac{4bc_1 - 8c_2}{2-\mathrm{b}} + (2 - c_1 - c_2)(2+b)\right)\right\}, \tag{21}$$

where $\overset{=u}{r} = 2z_\delta\sigma_\alpha(2+b) + \frac{2(1-c_2)-b(1-c_1)}{2-b}$ and z_δ is the value of z satisfying $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{-x^2}dx = \delta$.

The smallest feasible value of δ is the one satisfying

$$z_\delta^u = \frac{8 + 4c_1 - 12c_2 + 8bc_1 - 8\mathrm{b}c_2 - 2\mathrm{b}^2 + 3\mathrm{b}^2c_1 - \mathrm{b}^2c_2}{-2(4-\mathrm{b}^2)(3\sigma + 2\sigma b)}. \tag{22}$$

Proof) See Appendix 3.

The constraint in (P-U) is to guarantee the profitability of terminal operators with a certain risk level, δ. On the other hand, the risk level may not be lowered to such a level that the value of f becomes negative. The limitation in the risk level is represented by (22).

Property 4.2: The problem of case U reduces to the problem of case QD when $\delta = 0.5$.

Proof) See Appendix 4.

4.2. Case where the port investor has uncertain demand information but terminal operators know the cargo demand (case A)

This section assumes that the cargo amount of the port is uncertain to the port investor when the parameters of the rental fee are determined, while it is known to the container terminal operators. This case will be called the case with asymmetric information on cargo demand (Case A) in this paper. Considering the uncertainty in the cargo demand of the port, the relationship between the cargo amount and the terminal handling charge can be expressed as follows (Klemperer and Meyer 1986; Kultti and Niinimäki 1998; Chen, Lin and Liu 2017):

$$p_1 = 1 + \beta - q_1 - bq_2 \tag{23}$$

$$p_2 = 1 + \beta - q_2 - bq_1, \tag{24}$$

where β is a random variable to the port investor, which follows a normal distribution $N(0, \sigma_\beta^2)$. On the other hand, β is a constant value to the terminal operators because it is known to them.

The details to normalise the above equations are provided in Appendix 7. Because all the informations are known to terminal operators, they attempt to maxmimize their deterministic profit functions which have no random variable as follows:

(T-A)

$$\max_{q_i} \pi_i(f,\ r) = p_i q_i - (c_i + r)q_i - f \quad \text{for } i = 1, 2, \tag{25}$$

subject to $q_i \geq 0,\ i = 1,\ 2,$

and constraints (23) and (24).

The profit function for container terminal operators can be written as

$$\pi_1(f,\ r) = (1 + \beta - q_1 - bq_2 - c_1 - r)q_1 - f,$$

$$\pi_2(f,\ r) = (1 + \beta - q_2 - bq_1 - c_2 - r)q_2 - f.$$

When differentiating the above two profit functions with respect to q_i,

$$\frac{\partial \pi_1}{\partial q_1} = (1 + \beta - bq_2 - c_1 - r) - 2q_1$$

$$\frac{\partial \pi_2}{\partial q_2} = (1 + \beta - bq_1 - c_2 - r) - 2q_2.$$

From the first order condition that $\frac{\partial \pi_1}{\partial q_1} = \frac{\partial \pi_2}{\partial q_2} = 0$, the optimal q_1^{a*} and q_2^{a*} can be obtained as follows:

$$q_1^{a*} = \frac{1 - r + \beta}{2 + b} + \frac{bc_2 - 2c_1}{4 - b^2} \tag{26}$$

$$q_2^{a*} = \frac{1 - r + \beta}{2 + b} + \frac{bc_1 - 2c_2}{4 - b^2} \quad (\leq q_1^{a*}), \tag{27}$$

which are the decisions by the terminal operators.

For a given value of β, the minimum profit function for container terminal operator i may be expressed as $\pi_i^{a*}(f,\ r) = (q_i^{a*})^2 - f$.

Because β is a random variable in the long term, the long-term expected profit of container terminal operator i considering the uncertainty of β becomes $E_\beta[\pi_i^{a*}(f,\ r)] = E_\beta[(q_i^{a*})^2 - f]$. For example, $E_\beta[(q_2^{a*})^2 - f] = E_\beta\left[\left(\frac{1 - r + \beta}{2 + b} + \frac{bc_1 - 2c_2}{4 - b^2}\right)^2 - f\right] = \frac{Var(\beta)}{(2 + b)^2} + (q_2^{d*})^2 - f.$

Note that, from $\mathrm{Var}(\beta) = E[\beta^2] - (E[\beta])^2$, we can derive that $E[\beta^2] = \mathrm{Var}(\beta)$ with $\mathrm{E}[\beta] = 0$.

The revenue-maximizing problem of the port investor in case A may be expressed as

(P-A)

$$\max_{r,f} E_\beta(Z) = E_\beta[2f + r(q_1^{a*} + q_2^{a*})],$$

subject to $p[\pi_i^{a*}(f,\ r) \geq 0] \geq 1 - \delta$ for $i = 1$ and 2.

Property 4.3: For the problem of (P-A), the optimal parameters of the fixed and variable rental fee, (f^{a*}, r^{a*}), becomes

$$f^{a*} = \left(\frac{\sigma_\beta z_\delta}{2 + b} + \frac{1 - r}{2 + b} + \frac{bc_1 - 2c_2}{4 - b^2}\right)^2 \tag{28}$$

and

$$r^{a*} = \min\left\{\overset{=a}{r},\ \frac{-\sigma_\beta z_\delta}{1+\mathrm{b}} + \left(\frac{(4+\mathrm{b}^2)c_2 - (4+4\mathrm{b}-\mathrm{b}^2)c_1 + 4b - 2\mathrm{b}^2)}{4(1+\mathrm{b})(2-\mathrm{b})}\right)\right\}. \tag{29}$$

where $\overset{=a}{r} \equiv \sigma_\beta z_\delta + 1 + \dfrac{bc_1 - 2c_2}{2-\mathrm{b}}$.

The smallest feasible value of δ is the one satisfying $z^a_\delta = \dfrac{-2+\mathrm{b}-bc_1+2c_2}{\sigma_\beta(2-\mathrm{b})}$.

Proof) See Appendix 5.

4.3. Case where terminal operators have a smaller uncertainty on cargo demand than the port investor (case P)

In this case, both terminal operators and the port investor have uncertain information on the cargo demand but the uncertainty of the port investor is lager than that of terminal operators, which is called the case with 'partially asymmetric (P)' information in this paper. In order to express the difference in the uncertainty between the port investor and terminal operators, two random variables are introduced. The value of the first random variable, β, is known to terminal operators but it is an unknown random variable to the port investor, which follows a normal distribution $N(0,\ \sigma^2_\beta)$. And α represents the unknown random variable to both terminal operators and the port authority, which follows $N(0,\ \sigma^2_\alpha)$. Thus, the uncertainty of the cargo demand to the port investor is larger than that to terminal operators.

Then, the demand-price relationships are represented as follows:

$$p_1 = 1 + \beta + \alpha - q_1 - bq_2, \tag{30}$$

$$p_2 = 1 + \beta + \alpha - q_2 - bq_1, \tag{31}$$

Then, the profit function for container terminal operators can be written as

$$\pi_1(f,\ r) = (1 + \beta + \alpha - q_1 - bq_2 - c_1 - r)q_1 - f,$$

$$\pi_2(f,\ r) = (1 + \beta + \alpha - q_2 - bq_1 - c_2 - r)q_2 - f.$$

Because β is known to terminal operators, they have the following problems:

(T-P)

$\max\limits_{q_i} E_\alpha[\pi_i(f,\ r)] = E_\alpha[p_iq_i - (c_i + r)q_i - f]$ for $i = 1, 2,$

subject to $q_i \ge 0,\ \ i = 1,\ 2,$ and constraints (30) and (31).

The expected profit for container terminal operators to maximise can be written as

$$E_\alpha[\pi_1(f,\ r)] = (1 + \beta - q_1 - bq_2 - c_1 - r)q_1 - f,$$

$$E_\alpha[\pi_2(f,\ r)] = (1 + \beta - q_2 - bq_1 - c_2 - r)q_2 - f.$$

When differentiating the above two profit functions with respect to q_i,

$$\frac{\partial E_\alpha[\pi_1(f,\ r)]}{\partial q_1} = (1 + \beta - bq_2 - c_1 - r) - 2q_1,$$

$$\frac{\partial E_\alpha[\pi_2(f,\ r)]}{\partial q_2} = (1 + \beta - bq_1 - c_2 - r) - 2q_2.$$

From the first order condition that $\dfrac{\partial \pi_1}{\partial q_1} = \dfrac{\partial \pi_2}{\partial q_2} = 0$, the optimal q_1^{p*} and q_2^{p*} can be obtained as follows: $q_1^{p*} = \frac{1-r+\beta}{2+b} + \frac{bc_2 - 2c_1}{4-b^2}$, $q_2^{p*} = \frac{1-r+\beta}{2+b} + \frac{bc_1 - 2c_2}{4-b^2} (\leq q_1^{p*})$, which are the decisions by the terminal operators. By the same derivation in section 4.2, we can find the long-term expected profit of container terminal operator i considering the uncertainty of β becomes

$$E_\beta[E_\alpha[\pi_i^{p*}(f,\ r)]] = \frac{Var(\beta)}{(2+b)^2} + (q_2^{d*})^2 - f.$$

The revenue-maximizing problem of the port investor may be expressed as

(P–P)

$$\max_{r,f} E_{(\alpha,\beta)}(Z) = E_{(\alpha,\beta)}[2f + r(q_1^{p*} + q_2^{p*})]$$

subject to $P[\pi_i^{a*}(f,\ r) \geq 0] \geq 1 - \delta$ for $i = 1$ and 2.

The optimal values of r and f for problem (P–P) may be found by a two-dimensional search procedure, which is introduced in Appendix 6.

5. Numerical experiments

5.1. Sensitivity analysis of the optimal solutions for the case with a deterministic cargo demand (case QD)

To explore how the service substitution degree and variable cost affect the optimal solutions, including q_i^{d*}, π_i^{d*}, f^{d*}, r^{d*}, and Z^*, two groups of experiments were performed for the quantity competition model: experiments in which b varied; and those in which c_2/c_1 varied.

In the first group of experiments, the parameters, c_1 and c_2, were fixed to 0.1 and 0.2, respectively. The value of b ranged from 0.1–0.9. Table 2 shows that the optimal cargo amount of terminal operators 1 and 2 decreases monotonically with increasing value of service substitution. This is because, with a larger value of b, the cargo amount of a container terminal operator has a higher negative impact on the terminal handling charge of the other operator, the cargo amounts of both operators are forced to stay in lower levels in the Nash Equilibrium. The values of f^{d*} and Z^* decrease monotonically with increasing value of b. The value of r^{d*} increases monotonically with increasing value of b.

Container terminal operator 2 has a higher decreasing rate in the optimal cargo amount. Note that the optimal profit of terminal operator container 2 always remains at zero. This is because the port investor can increase its revenue by increasing the value of f until the profit of terminal operator container 2, which is less competitive than operator 1, reaches zero.

In the second group of experiments, assuming a fixed value of $b = 0.5$, c_2/c_1 varied with the constraint that $c_2 + c_1 = 0.3$. Table 3 shows that the optimal cargo amount of container terminal operator 1 decreases monotonically with decreasing c_2/c_1 and the optimal cargo amount of container terminal operator 2 increases monotonically. This is because, as c_2/c_1 decreases from 29.0–2.33, the competitiveness of terminal operator 2 becomes similar to that of container terminal operator 1. The optimal profit of terminal operator 2 always equals zero and the optimal profit of terminal

Table 2. Sensitivity analysis of the optimal solutions for various values of b in case QD.

	b								
Solution	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
q_1^{d*}	0.389	0.359	0.334	0.313	0.294	0.279	0.266	0.255	0.245
q_2^{d*}	0.336	0.303	0.275	0.250	0.228	0.208	0.189	0.171	0.154
π_1^{d*}	0.038	0.037	0.036	0.035	0.035	0.035	0.035	0.035	0.036
π_2^{d*}	0	0	0	0	0	0	0	0	0
f^{d*}	0.113	0.092	0.076	0.063	0.052	0.043	0.036	0.029	0.024
r^{d*}	0.089	0.122	0.150	0.175	0.197	0.217	0.236	0.254	0.271
Z^*	0.290	0.265	0.242	0.223	0.207	0.192	0.179	0.167	0.156

Table 3. Comparison of the revenue of the port investor for various values of c_2/c_1 in case QD.

	c_2/c_1								
Solution	29.00	14.00	9.00	6.50	5.00	4.00	3.29	2.75	2.33
q_1^{d*}	0.314	0.312	0.310	0.308	0.306	0.303	0.301	0.299	0.297
q_2^{d*}	0.128	0.139	0.150	0.161	0.172	0.183	0.194	0.206	0.217
π_1^{d*}	0.083	0.078	0.074	0.069	0.064	0.058	0.053	0.047	0.041
π_2^{d*}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
f^{d*}	0.016	0.019	0.023	0.026	0.030	0.034	0.038	0.042	0.047
r^{d*}	0.297	0.286	0.275	0.264	0.253	0.242	0.231	0.219	0.208
Z^*	0.164	0.168	0.172	0.176	0.180	0.185	0.190	0.195	0.201

operator 1 decreases monotonically. In addition, the values of f^{d*} and Z^* increase monotonically with decreasing c_2/c_1. Note that as the revenue of the port investor increases, the total profit of the container terminal operators decreases. As the cost functions of both container terminal operators become similar to each other, it becomes easier for the port investor to set the parameters of the revenue sharing scheme to absorb the profits of container terminal operators at the same time than when the cost functions of both operators are considerably different from each other.

5.2. Comparing the quantity and price competition models with a deterministic cargo demand

The results from the quantity competition model and those from the price competition model were compared with each other for various values of b. Generally, the price competition model showed higher revenues than the quantity competition model. The revenue decreased with increasing value of b for both the quantity and price competition models. The gap in revenue between the quantity and price competition models increased with increasing b, as listed in Table 4.

5.3. Comparing the results by this study with those of previous studies for the case with a deterministic cargo demand

5.3.1. Comparing the results for the quantity competition model (case QD)

Consider the objective function of the port authority of (9): $Z = 2f + r(q_1^{d*} + q_2^{d*})$. Chen and Liu (2014) proposed the expressions of the optimal r and f as follows:

$$r^{d*} = \frac{1}{(4b+6)}\left\{(2+2b) + \frac{c_2 b^2}{(2-b)} - \frac{(4+2b-b^2)c_1}{(2-b)}\right\}, \tag{32}$$

and

$$f^{d*} = (q_2^{d*})^2/2. \tag{33}$$

Table 4. Gap in revenue between the quantity and price competition models.

	Reveune		
b	Quantity competition model (case QD)	Price competition model (case PD)	*Gap* (%)
0.1	0.2904	0.2906	0.06
0.2	0.2645	0.2652	0.26
0.3	0.2425	0.2439	0.59
0.4	0.2234	0.2259	1.08
0.5	0.2068	0.2104	1.78
0.6	0.1920	0.1972	2.74
0.7	0.1787	0.1861	4.10
0.8	0.1667	0.1772	6.29
0.9	0.1558	0.1732	11.22

$Gap = 100*(Z_1^* - Z_2^*)/Z_2^*$ (%) (Z_1^*: revenue by price competition model; Z_2^*: revenue by quantity competition model).

Chen and Liu (2014) optimised r first assuming that f is constant, which results in (32). The value of f was then determined sequentially by increasing the value of f until the profit of either of the two terminals became zero, which arrives at (33). The above expressions are different from those in Property 2.1 of this study. This study optimised r and f simultaneously. Note that q_2^* again depends on the value of r, as shown in (4). Thus, f above is again dependent on the value of r. Therefore, the expressions of the optimal solutions in this study must give higher revenue to the port investor than those by Chen and Liu (2014).

Consider the constraints $\pi_1^{d*} \geq 0$ and $\pi_2^{d*} \geq 0$ of (P-QD). Considering $\pi_1^{d*} \geq \pi_2^{d*}$, the condition can be expressed as $\pi_2(f, r) = (q_2^*)^2 \geq 0$, which was expressed incorrectly as $\pi_2(f, r) = (q_2^*)^2 - f \geq f$ by Chen and Liu (2014).

In the first numerical experiment, the values of the input parameters (Table 5), including b, c_1 and c_2 are from Chen and Liu (2014). Tables 6 and 7 compare the solutions provided by Chen and Liu (2014), using the input data shown in Table 5, with those in this study. From Tables 6 and 7, the solutions by this study provide higher revenues to the port investor than those reported by Chen and Liu (2014): by 11.57% and 0.043% for examples 1 and 2, respectively, in Table 5.

In the second group of experiments, first, for fixed values of c_1 and c_2, the value of b varied. The revenues from this study were larger than those from the previous study for all values of b. Table 8 lists the gap in the revenue between the two studies. The gap ranged between 13.77% and 36.52% for various values of b.

Next, c_2/c_1 varied with a fixed value of their summation ($c_2 + c_1 = 0.3$) and of b. The revenues by Chen and Liu (2014) were compared with those by this study for various values of c_2/c_1. The revenue obtained by this study was always higher than that by Chen and Liu (2014). Table 8 shows that the minimum and maximum gap is 8.06% and 21.26%, respectively.

5.3.2. Comparing the results for the price competition model (case PD)

This section compares the results obtained by Chen, Lin, and Liu (2017) and those by this study for the Bertrand model. Regarding the differences between the optimal solutions in Chen, Lin, and Liu (2017) and those in case PD of this study, an explanation similar to those at the beginning of section 5.3.1 may be applied. In the following experiments, the values of parameters were fixed to $c_1 = 0.1$ and $c_2 = 0.2$.

Table 5. Experimental parameter setting of Chen and Liu (2014).

	Parameters		
Numerical examples	b	c_1	c_2
1	0.99999	0.1	0.2
2	0.5	0.1	0.5

Table 6. Comparison between the solutions in Chen and Liu (2014) and those in this study of example 1.

	Optimal solutions						
Methods	r^{d*}	q_1^{d*}	q_2^{d*}	π_1^{d*}	π_2^{d*}	f^{d*}	Z^*
Chen and Liu (2014)	0.37	0.21	0.11	0.03805	0.00605	0.00605	0.1305
This study	0.2875	0.2375	0.1375	0.03750	0.0	0.018906	0.1456

Table 7. Comparison between the solutions in Chen and Liu (2014) and those in this study of example 2.

	Optimal solutions						
Methods	r^{d*}	q_1^{d*}	q_2^{d*}	π_1^{d*}	π_2^{d*}	f^{d*}	Z^*
Chen and Liu (2014)	0.3458	0.275	0.0083	0.07559	0.000035	0.000035[§]	0.0981056
This study	0.3389	0.278	0.0111	0.07703	0.0	0.0001235	0.0981481

[§]In Chen and Liu (2014), $f^{d*} = 0.00006889$, which is different from the value (0.000035) by $f^{d*} = (q_2^{d*})^2/2$ and $q_2^{d*} = 0.0083$. The latter was adopted in this table.

Table 8. Gap in the revenue between the two studies for various values of b and c_2/c_1.

b	*Gap* (%)	c_2/c_1	*Gap* (%)
0.1	36.52	29	8.06
0.2	32.52	14	9.44
0.3	29.04	9	10.91
0.4	25.96	6.5	12.46
0.5	23.18	5	14.09
0.6	20.63	4	15.79
0.7	18.24	3.29	17.56
0.8	15.97	2.75	19.38
0.9	13.77	2.33	21.26
$c_1 = 0.1$, $c_2 = 0.2$		b = 0.5, $c_2 + c_1 = 0.3$	

$Gap = 100*(Z_1^* - Z_2^*)/Z_2^*$ (%) (Z_1^*: revenue by this study; Z_2^*: revenue by Chen and Liu (2014))

Table 9. Gap between the revenues by Chen, Lin, and Liu (2017) and those in case PD by this study.

b	Gap (%)
0.1	36.13
0.2	31.14
0.3	26.21
0.4	21.36
0.5	16.58
0.6	11.90
0.7	7.37
0.8	3.17
0.9	0.12

$Gap = 100*(Z_1^* - Z_2^*)/Z_2^*$ (%)
(Z_1^*: revenue by this study; Z_2^*: revenue by Chen and Liu (2014))

Table 10. Optimal solutions of two cases with symmetric information on cargo demand for example 1.

	Demand known to all (case QD)	Demand uncertain to all (case U)				
δ	-	0.01	0.05	0.08	0.10	0.15
q_1^*	0.2375	0.3330	0.2650	0.2410	0.2290	0.2115
q_2^*	0.1375	0.2330	0.1650	0.1410	0.1290	0.1115
π_1^*	0.0375	0.1109	0.0702	0.0581	0.0524	0.0439
π_2^*	0.0000	0.0543	0.0272	0.0199	0.0166	0.0116
f^*	0.0189	0.0000	0.0000	0.0000	0.0000	0.0008
r^*	0.2875	0.0010	0.2050	0.2770	0.3130	0.3655
Z	0.1456	0.0006	0.0882	0.1058	0.1121	0.1197
Total profit	0.0375	0.1652	0.0974	0.0780	0.0691	0.0555

The optimal solution by this study showed higher revenues than the study of Chen, Lin, and Liu (2017) for all values of b. Table 9 also shows that the gaps of the revenue obtained by this study and that Chen, Lin, and Liu (2017) decreases with increasing value of b.

5.4. Comparing cases with a deterministic demand and an uncertain demand

This section compares the optimal solutions among the case with a deterministic cargo demand (case QD) and the three cases with an uncertain cargo demand. The following experiments were conducted using two groups of parameters: (example 1) $b = 0.99999$, $c_1 = 0.1$, $c_2 = 0.2$, and $\sigma = 0.1$; (example 2) $b = 0.5$, $c_1 = 0.1$, $c_2 = 0.5$, and $\sigma = 0.1$.

Table 10 lists the optimal solutions and objective values of two cases (cases QD and U) with symmetric information on cargo demand for example 1 at various levels of maximum risk of loss for the terminal operators, δ. Note that the maximum risk of loss for the terminal operators, δ, plays a role in restricting the probability of the terminal operators having a loss. Therefore, to guarantee a lower risk of loss, the port investor needs to pay the cost of sacrificing its revenue.

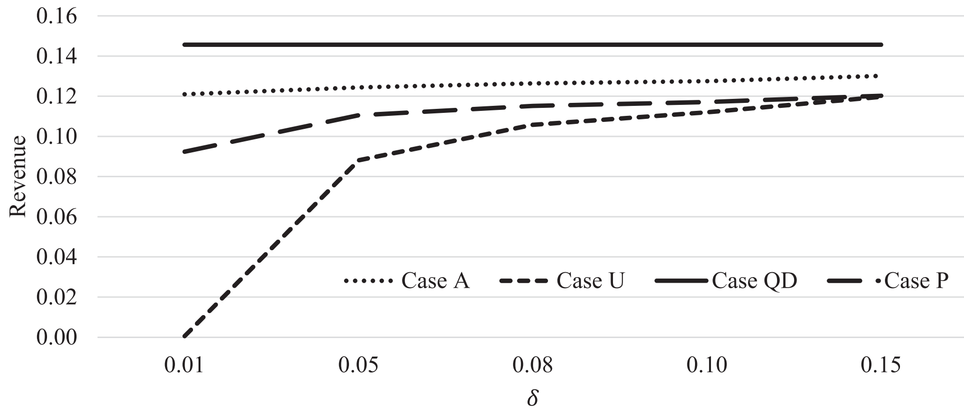

Figure 1. Revenue of the port investor in four cases on cargo demand for example 1.

Figures 1 and 2 show the trends of the revenue of the port investor and the total profit of terminal operators when the maximum risk of loss for the terminal operators, δ, changes. As shown in Figure 1 and Table 10, in case U, the revenue of the port investor as well as the values of f^* and r^* decrease as the maximum risk of loss decreases. On the other hand, as shown in Figure 2, the total profit of the two terminal operators increases.

Table 11 shows optimal solutions and objective values in cases A and P for example 1 for various levels of the maximum risk of loss. As the maximum risk of loss decreases, the fixed fee decreases in both cases, while the unit rental fee showed different trends with each other. Figure 1 shows that the revenue decreases as the maximum risk of loss decreases in cases A and P, as in case U. Figure 2 shows that the total profit of terminal operators increases as the maximum risk of loss decreases in case P as in case U, while the curve of the total profit curve of case A does not follow the general trend of decreasing. For example 2, for all the three cases with uncertain demand (cases U, A, and P), as the maximum risk of loss decreases, the revenue decreases and the total profit of terminal operators increases. which are shown in Tables 12 and 13.

Comparing three cases with different assumptions on the demand in Figures 1 and 2, case QD showed the highest (expected) revenue to the port investor and the lowest (expected) profits to every participating terminal operator. Case U showed the lowest (expected) revenue to the port investor. Cases A and P showed the (expected) revenue of the port authority in-between the two extreme cases. Note that when the maximum risk of loss of the terminal operators, δ, increases,

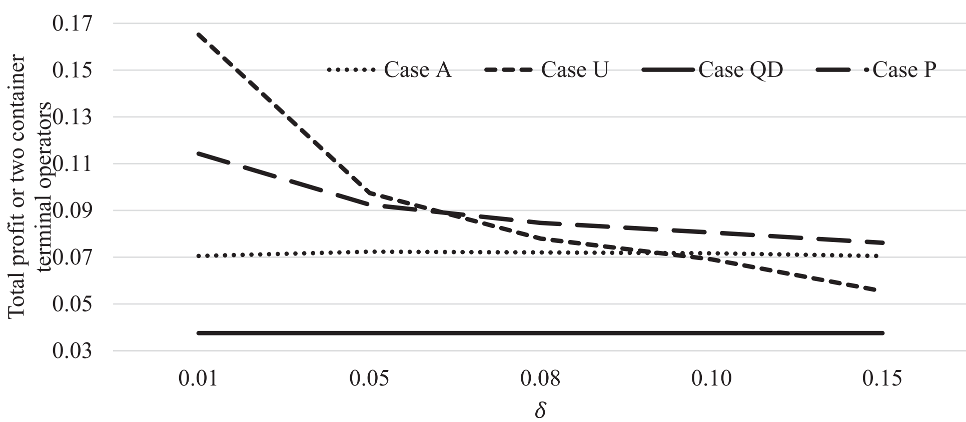

Figure 2. Total profit of terminal operators in four cases on cargo demand for example 1.

Table 11. Optimal solutions of two cases with asymmetric information on cargo demand for example 1.

	Demand known to terminal operators but uncertain to the port investor (case A)					Demand uncertainty is larger for the port investor than for terminal operators (case *P*)				
δ	0.01	0.05	0.08	0.10	0.15	0.01	0.05	0.08	0.10	0.15
q_1^*	0.1987	0.2100	0.2140	0.2160	0.2202	0.2600	0.2322	0.2211	0.2150	0.2119
q_2^*	0.0987	0.1100	0.1140	0.116	0.1202	0.1600	0.1322	0.1211	0.1150	0.1119
π_1^*	0.0501	0.0522	0.0524	0.0524	0.0523	0.0781	0.0644	0.0594	0.0568	0.0543
π_2^*	0.0204	0.0202	0.0196	0.0192	0.0182	0.0361	0.0280	0.0252	0.0238	0.0219
f^*	0.0004	0.0030	0.0045	0.0053	0.0073	0.0000	0.0000	0.0000	0.0000	0.0012
r^*	0.4040	0.3700	0.3580	0.3520	0.3395	0.2200	0.3035	0.3366	0.3549	0.3644
Z	0.1210	0.1245	0.1264	0.1275	0.1302	0.0924	0.1106	0.1152	0.1171	0.1203
Total profit	0.0705	0.0724	0.0720	0.0717	0.0705	0.1143	0.0925	0.0847	0.0806	0.0762

Table 12. Optimal solutions of two cases with symmetric information on cargo demand for example 2.

	Demand known to all (Case QD)	Demand uncertain to all (Case U)				
δ	-	0.01	0.05	0.08	0.10	0.15
q_1^*	0.2778	-*	-	0.4077	0.3957	0.3707
q_2^*	0.0111	-	-	0.1410	0.1290	0.1040
π_1^*	0.0770	-	-	0.1662	0.1566	0.1374
π_2^*	0.0000	-	-	0.0199	0.0166	0.0108
f^*	0.000123	-	-	0.0000	0.0000	0.0000
r^*	0.3389	-	-	0.0142	0.0442	0.1067
Z^*	0.0981	-	-	0.0078	0.0232	0.0506
Total profit	0.0770	-	-	0.1861	0.1732	0.1482

* - indicates that there does not exist a feasible solution

Table 13. Optimal solutions of two cases with asymmetric information on cargo demand for example 2.

	Demand known to terminal operators but uncertain to the port investor (case A)					Demand uncertainty is larger for the port investor than for terminal operators (case *P*)				
δ	0.01	0.05	0.08	0.10	0.15	0.01	0.05	0.08	0.10	0.15
q_1^*	0.3599	0.3327	0.3231	0.3183	0.3083	-*	-	0.4119	0.4046	0.3883
q_2^*	0.0932	0.0660	0.0564	0.0516	0.0416	-	-	0.1452	0.1379	0.1216
π_1^*	0.1455	0.1267	0.1204	0.1173	0.1110	-	-	0.1848	0.1789	0.1660
π_2^*	0.0247	0.0204	0.0192	0.0187	0.0177	-	-	0.0363	0.0342	0.0300
f^*	0.0000	0.0000	0.0000	0.0000	0.0000	-	-	0.0000	0.0000	0.0000
r^*	0.1337	0.2017	0.2257	0.2377	0.2627	-	-	0.0037	0.0219	0.0626
Z^*	0.0606	0.0804	0.0856	0.0879	0.0919	-	-	0.0021	0.0119	0.0319
Total profit	0.1702	0.1470	0.1396	0.1360	0.1288	-	-	0.2211	0.2131	0.1959

* - indicates that there does not exist a feasible solution

the profits of the terminal operators and the revenue of the port investor of cases U, A, and P approach those of case QD.

Figures 3–5 compare the revenue, the total profit of terminal operators, and the total welfare utilising results in Tables 10–13. Values in the figures are normalised so that the values of case QD become 1.0. Figure 3 compares the revenue of the port investor among various cases. The port investor may collect the highest revenue in case QD in which the port investor takes advantage of utilising known demand information as the leader, while the revenue was lowest in case U. Figure 4 shows that the total profit of terminal operators is lowest in case QD, while it is high in cases A and P in which terminal operators take advantage of utilising the knowledge on the demand.

Figure 5 compares the total welfare of the port which is the sum of the revenue of the port investor and the profits of terminal operators, which indicates how good the decisions are from the viewpoint of the entire port. Cases A and P showed relatively higher total welfares than cases QD and U.

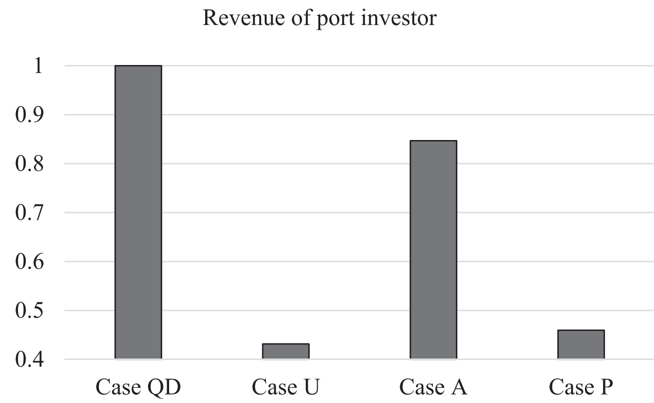

Figure 3. Comparison of the revenue of the port investor among various cases.

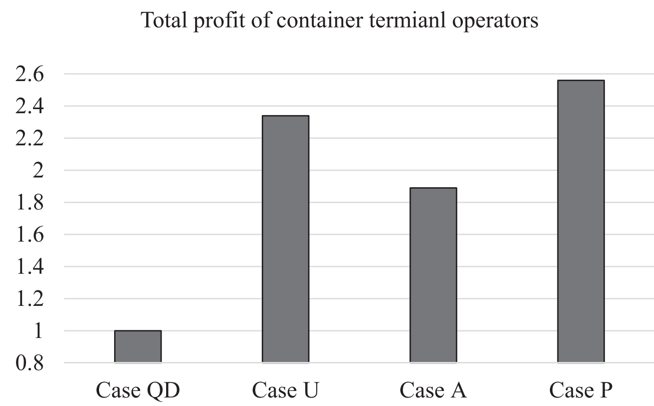

Figure 4. Comparison of the total profit of terminal operators among various cases.

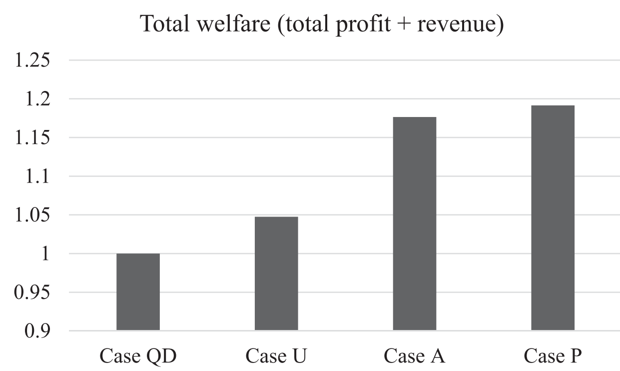

Figure 5. Comparison of the total welfare of the port among various cases.

5.5. Managerial implications and additional discussions about the models

This study assumed that the port investor aims to maximise the total revenue that is paid by the container terminal operators. Some managerial implications from the numerical experiments for the case with a deterministic demand are as follows: the price competition model produces better performance than that of the quantity competition model in terms of the revenue of the port investor.

As the service substitution parameter becomes smaller, which means that the influence of the throughput of one terminal operator on the price of the other operator becomes smaller, and the operation cost parameters of the two terminals become similar, the revenue of the port investor becomes larger for both the price and quantity competition models.

Through the experiments on the case with the cargo demand uncertainty, the following interesting policy implications were found: (1) the cargo demand uncertainty decreases the revenue of the port investor but tends to increase the profit of the terminal operators; (2) the revenue of the case with a deterministic cargo demand is highest at the cost of the lowest profits of terminal operators; (3) the case with a demand uncertain to all participants reduces the revenue of the port investor further than cases with an asymmetrical uncertain demand; and (4) the level of the risk of loss for terminal operators has a significant impact on the revenue of the port investor and the profits of terminal operators; (5) the cases with asymmetric information on cargo demand show relatively higher total welfares than case D, which means that information on the cargo demand does not contribute to the improvement of the port welfare in this leader-follower game model.

All the parameters and variables in this study were normalised. This section introduces how to convert the general models to normalised models. The relationship between the original parameters and variables and the normalised parameters and variables are explained in the following. The linear demand model used in this study follows the studies reported by Singh and Vives (1984) and Dong, Zheng, and Lee (2018), which were modelled as follows: $\bar{p}_1 = \tau_1 - \theta\bar{q}_1 - \omega\bar{q}_2$ and $\bar{p}_2 = \tau_2 - \theta\bar{q}_2 - \omega\bar{q}_1$, where τ_1 and τ_2 denote the maximum price of each terminal that the customers want to pay. The parameters, θ, and ω, indicate the substitution parameters of the two container terminal operators. In this study, it was assumed that $\theta \geq \omega > 0$, according to Dong, Zheng, and Lee (2018). The units of τ_1 and τ_2 are US \$. The units of θ and ω are US \$ per TEU. The units of $\bar{p}_i$ and $\bar{q}_i$, which are the original price and cargo amount of terminal i, are US \$ and TEU, respectively. Because container terminals 1 and 2 are located in the same port, it was assumed that $\tau_1 = \tau_2 = \tau$ (Dong, Huang, and Ng 2016). The following linear relationships may express the price and cargo amount of the terminals:

$$\bar{p}_1 = \tau - \theta\bar{q}_1 - \omega\bar{q}_2 \text{ and } \bar{p}_2 = \tau - \theta\bar{q}_2 - \omega\bar{q}_1.$$

By assuming $\varphi = \tau/\theta$, $q_1 = \frac{\bar{q}_1}{\varphi}$, $q_2 = \frac{\bar{q}_2}{\varphi}$, $p_1 = \frac{\bar{p}_1}{\theta\varphi}$, and $b = \frac{\omega}{\theta}$, the above equations can be normalised as follows: $p_1 = 1 - q_1 - bq_2$ and $p_1 = 1 - q_1 - bq_2$.

The profit functions for the container terminal operators can be expressed as $\bar{\pi}_1 = \bar{p}_1\bar{q}_1 - (\bar{c}_1 + \bar{r})\bar{q}_1 - \bar{r}$ and $\bar{\pi}_2 = \bar{p}_2\bar{q}_2 - (\bar{c}_2 + \bar{r})\bar{q}_2 - \bar{f}$, where $\bar{r}$ and $\bar{f}$ are the original unit rental fee and the original fixed rental fee, respectively. The relationship between the original and normalised parameters or variables are expressed as $c_1 = \frac{\bar{c}_1}{\theta\varphi^2}$, $c_2 = \frac{\bar{c}_2}{\theta\varphi^2}$, $r = \frac{\bar{r}}{\theta\varphi^2}$, $f = \frac{\bar{f}}{\theta\varphi^2}$, $\pi_1 = \frac{\bar{\pi}_1}{\theta\varphi^2}$, and $\pi_2 = \frac{\bar{\pi}_2}{\theta\varphi^2}$. Appendix 7 provides the detail derivations not only for the case with a deterministic cargo demand but also with an uncertain cargo demand.

6. Conclusions

To support port investors to design concession contracts, this study proposed new expressions or procedures for determining fixed and variable rental fees in concession contracts. Both cases with a deterministic cargo demand and with uncertain demand were analysed. For the deterministic demand case, both the quantity competition and the price competition models were analyzed. Sensitive analyses were conducted to determine how the service substitution degree and variable cost affect the optimal solutions. Finally, to evaluate the optimal solutions for the cases with a deterministic cargo demand, which are derived in this study, they were compared with the optimal solutions derived by previous studies (Chen and Liu 2014; Chen, Lin, and Liu 2017). The numerical experiment showed that the proposed solution gives higher revenues of the port investor than those given by previous studies.

Three cases with an uncertain cargo demand were also analysed: case with demand information uncertain to all the participants; case with a deterministic demand for terminal operators and an uncertain demand for the port investor; case where the port investor has a larger uncertainty on demand than terminal operators. According to the numerical experiment, the revenue of the port investor is the lowest when information on the cargo demand is not known to all participants of the game, while it was highest when the demand information is known to all deterministically. The results were in-between the two extreme cases when the information on the cargo demand is known only to the terminal operators or the uncertainty is larger to the port investor than to terminal operators. It is interesting that the total welfare in a port and the total profit of terminal operators were lowest, when the cargo demand is known to all the participants.

Future studies can be considered in the following aspects: (1) this study assumed that container terminals compete with each other only by using the cargo amount or the terminal handling charge, which have a linear relationship with each other. However, the competitiveness of a container terminal may not be explained only by the handling charge but also by other factors such as the productivity, the connectivity, and so on, which needs to be included in future studies. In addition, because some conclusions in this study may come from the assumption on the linearity of the demand function, which is oversimplified for the application to the practice and needs to be relaxed; (2) it is necessary to study on the design of new concession contract schemes which are different from that in this study and effective to improve various performance measures such as the total profit of terminals, the revenue of the investor, and the social welfare; (3) according to the numerical experiment in this study, it was found that the two-staged game with a known cargo demand provides the lowest welfare to the entire port. This means that the reduction of uncertainty in cargo demand does not contribute to the improvement of the total welfare of a port. Instead, it will be meaningful to study how a port authority or a government agency can attract terminal operators to move to the decisions which maximise the total welfare of a port as well as individual participants' interest.

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Appendices

Appendix 1. Proof of Property 2.1

From $\pi_i^{d*}(f,r)=(q_i^{d*})^2-f$ for $i=1,\ 2$, and $\pi_1^{d*}(f,r)>\pi_2^{d*}(f,r)$, the constraints in (P-QD), $\pi_1^{d*}\geq 0$ and $\pi_2^{d*}\geq 0$, may be rewritten as $(q_2^{d*})^2\geq f$. Because Z for a given value r increases with increasing f, $f^{d*}=(q_2^{d*})^2$. Thus, (P-QD) becomes $\max_r Z=2(q_2^{d*})^2+r(q_1^{d*}+q_2^{d*})$ subject to $0\leq r\leq \bar{r}$. $\frac{\partial^2 Z}{\partial r^2}=\frac{-4(1+b)}{(2+b)^2}<0$, which implies that Z is concave. From the first order condition for the optimal solution, $\frac{\partial Z}{\partial r}=0$, $r^{d*}=\frac{(4+b^2)c_2-(4+4b-b^2)c_1+4\mathrm{b}-2b^2}{4(1+b)(2-b)}$.

For r^{d*} to be feasible, it should hold that $0\leq r^{d*}\leq\bar{r}$. Therefore, $r^{d*}\geq 0$.

$$r^{d*}=\frac{(4+b^2)c_2-(4+4b-b^2)c_1+4b-2b^2}{4(1+b)(2-b)}>\frac{(4+b^2)c_1-(4+4b-b^2)c_1+4\mathrm{b}-2b^2}{4(1+b)(2-b)}=\frac{b(1-c_1)}{2(1+b)}>0.$$

Note that the first inequality comes from c_2.

Note that $\bar{r}-r^{d*}=-\frac{(b+2)(2b-2c_1+6c_2-3bc_1+bc_2-4)}{4(-b^2+b+2)}$. Because $b+2>0$ and $-b^2+b+2>0$, $\bar{r}-r^{d*}=0$ indicates $2b-2c_1+6c_2-3bc_1+bc_2-4=0$, which gives $c_2=\hat{c}_2=\frac{4+3bc_1+2c_1-2b}{6+b}$. $\bar{c}_2-\hat{c}_2=\frac{(b^2-4)(c_1-1)}{2(b+6)}>0$, which means that $\hat{c}_2$ is a tighter upper bound of c_2 than $\bar{c}_2$. ■

Appendix 2. Proof of Property 3.1

From $\pi_i^{d*}(f,r)=(1-b^2)(q_i^{d*})^2-f$ for $i=1,\ 2$, and $\pi_1^{d*}(f,r)>\pi_2^{d*}(f,r)$, the constraints in (P-PD), $\pi_1^{d*}\geq 0$ and $\pi_2^{d*}\geq 0$, may be rewritten as $(1-b^2)(q_2^{d*})^2\geq f$. Because Z for a given value r increases with increasing f, $f^{d*}=(1-b^2)(q_i^{d*})^2$. Thus, (P-PD) becomes $\max_r Z=2(1-b^2)(q_2^{d*})^2+r(q_1^{d*}+q_2^{d*})$ subject to $0\leq r\leq\bar{r}$.
$\frac{\partial^2 Z}{\partial r^2}=\frac{-4}{(1+b)(2-b)^2}<0$, which implies that Z is concave. From the first order condition for the optimal solution, $\frac{\partial Z}{\partial r}=0$, $r^{d*}=\frac{(4-3b^2)c_2-(4-b^2+4b)c_1+2b(2+b)}{4(2+b)}$.

From $(4-3b^2)>0$ and $c_2>c_1$,

$$\begin{aligned} r^{d*}&=\frac{(4-3b^2)c_2-(4-b^2+4b)c_1+2b(2+b)}{4(2+b)}\\ &\geq\frac{(4-3b^2)c_1-(4-b^2+4b)c_1+2b(2+b)}{4(2+b)}\\ &=\frac{2b^2(1-c_1)+4b(1-c_1)}{4(2+b)}>0. \end{aligned}$$

From the constraint, $r^{d*}\leq\bar{r}$, the following tighter upper bound of c_2, $\hat{c}_2$ may be obtained, where $\hat{c}_2=\frac{(8-8\mathrm{b}-2b^2+2b^3)+(4+4b-5b^2+b^3)c_1}{12-4b-7b^2+3b^3}$. $\bar{c}_2>\hat{c}_2$, whose proof is omitted here but provided in the supplementary document. Thus, r^{d*} satisfies the constraint $0\leq r^{d*}\leq\bar{r}$. ■

Appendix 3: Proof of Property 4.1

Because $\pi_1^{u*}(f,\ r)=(1+\alpha-q_1^{u*}-bq_2^{u*}-c_1-r)q_1^{u*}-f=(1-q_1^{u*}-bq_2^{u*}-c_1-r)q_1^{u*}+\alpha q_1^{u*}-f$ and $\pi_2^{u*}(f,\ r)=(1+\alpha-q_2^{u*}-bq_1^{u*}-c_2-r)q_2^{u*}-f=(1-q_2^{u*}-bq_1^{u*}-c_2-r)q_2^{u*}+\alpha q_2^{u*}-f$, the profit for container terminal operator i can be rewritten as follows: $\pi_i^{u*}(f,\ r)=(q_i^{u*})^2+\alpha q_i^{u*}-f$. Because $q_1^{u*}\geq q_2^{u*}$and $\pi_i^{u*}(f,\ r)$ increases with increasing q_i^{u*}, $\pi_1^{u*}\geq\pi_2^{u*}$. Thus, the constraint, $\mathrm{P}\{\pi_i^{u*}(f,\ r)\geq 0\}\geq 1-\delta$ for $i=1$ and 2, may be rewritten as $P(\alpha q_2^{u*}\geq f-(q_2^{u*})^2)\geq 1-\delta$, which is converted to $P\left(\frac{\alpha}{\sigma_\alpha}\geq\frac{f-(q_2^{u*})^2}{\sigma_\alpha q_2^{u*}}\right)\geq 1-\delta$. For a given value of r, the objective value of (P-U) increases with increasing f.

Thus, f^{u*} is the maximum f satisfying $P\left(\frac{\alpha}{\sigma_\alpha} \geq \frac{f-(q_2^{u*})^2}{\sigma_\alpha q_2^{u*}}\right) \geq 1-\delta$. Note that $\frac{\alpha}{\sigma_\alpha}$ follows the standard normal distribution. Thus, the value of f satisfying $z_\delta^u = \frac{f-(q_2^{u*})^2}{\sigma_\alpha q_2^{u*}}$ maximizes the revenue of the port investor, which may be rewritten as

$$f^{u*} = z_\delta \sigma_\alpha q_2^{u*} + (q_2^{u*})^2. \tag{A1}$$

From the condition that $f^{u*} \geq 0$, which is the same as $q_2^{u*}(z_\delta\sigma_\alpha + q_2^{u*}) \geq 0$. This inequality may be changed to $q_2^{u*} \geq -z_\delta\sigma_\alpha$. This inequality provides an upper bound on r,

$$\overline{\overline{r}}^u \equiv z_\delta\sigma_\alpha(2+b) + \frac{2(1-c_2)-b(1-c_1)}{2-b}. \tag{A2}$$

The revenue of the port investor becomes $E_\alpha(Z) = E[2f + r(q_1^{u*}+q_2^{u*})] = 2z_\delta\sigma_\alpha q_2^{u*} + 2(q_2^{u*})^2 + r(q_1^{u*}) + r(q_2^{u*})$, while $q_1^{u*} = \frac{1-r}{(1+b)(2-b)} + \frac{c_2 b - c_1(2-b^2)}{(1-b^2)(4-b^2)}$, $q_2^{u*} = \frac{1-r}{(1+b)(2-b)} + \frac{c_1 b - c_2(2-b^2)}{(1-b^2)(4-b^2)}$.

Note that $\frac{\partial^2 E_\alpha(Z)}{\partial r^2} = \frac{2(-1-b+b^2)}{\{(1+b)(2-b)\}^2} < 0$. Thus, $E_\alpha(Z)$ is concave. From the first order condition for the optimal solution, $\frac{\partial E_\alpha(Z)}{\partial r}$=0,

$$r^{u*} = \frac{-2z_\delta\sigma_\alpha(2+b)}{4+4b} + \frac{1}{4+4b}\left(\frac{(4+b^2)c_2 + (-4b-4+b^2)c_1}{2-b} + 2b\right). \tag{A3}$$

By $c_2 > c_1$, $r^{u*} > \frac{-2z_\delta\sigma_\alpha(2+b)}{4+4b} + \frac{1}{4+4b}\left(\frac{(4+b^2)c_1 + (-4b-4+b^2)c_1}{2-b} + 2b\right) = \frac{-2z_\delta\sigma_\alpha(2+b)}{4+4b} + \frac{2b(1-c_1)}{4+4b} > 0$. Thus, together with (A1), A(2), and (A3), the conclusion holds. ∎

Appendix 4: Proof of Property 4.2

Note that, considering that α follows $N(0, \sigma_\alpha^2)$, $P\{\pi_i^{u*}(f, r) \geq 0\} \geq 0.5$ is equivalent to $E_\alpha[\pi_i^{u*}(f, r)] \geq 0$, which may be converted to $\pi_i^{d*} \geq 0$ for $i=1$ and 2. Note also that $q_i^{u*} = q_i^{d*}$ for $i=1$ and 2 and so, from $q_2^{u*} = q_2^{d*} \geq 0$, the constraint, $r \leq \bar{r}$, should hold for the solution to be feasible. Thus, (P-U) can be rewritten as follows:

$$\max_{r,f} E_\alpha[Z] = E[2f + r(q_1^{d*}+q_2^{d*})] = 2f + r(q_1^{d*}+q_2^{d*})$$

subject to $0 \leq r \leq \bar{r}$, $\pi_i^{d*} \geq 0$ for $i=1$ and 2,
which is the same as (P-QD). Note that the optimal decisions by terminal operators are the same for the two cases. Thus, the conclusion holds. ∎

Appendix 5: Proof of Property 4.3

First, consider the constraint, $P[\pi_i^{a*}(f, r) \geq 0] \geq 1-\delta$ for all i. Considering $\pi_2^{a*}(f, r) < \pi_1^{a*}(f, r)$ and $\pi_i^{a*}(f, r) = (q_i^{a*})^2 - f$, this constraint may be rewritten as $P[(q_2^{a*})^2 \geq f] \geq 1-\delta$. Note that for a given value of β and r, $Z = 2f + r(q_1^{a*}) + r(q_2^{a*})$ increases with increasing f. Thus, f^{a*} is the maximum f satisfying $P[(q_2^{a*})^2 \geq f] \geq 1-\delta$ from which $P\left(\frac{\beta}{\sigma_\beta} \geq \frac{r-1+(2+b)\left(\sqrt{f}-\frac{bc_1-2c_2}{4-b^2}\right)}{\sigma_\beta}\right) \geq 1-\delta$. Because $\frac{\beta}{\sigma\beta}$ follows the standard normal distribution, $z_\delta \geq \frac{r-1+(2+b)\left(\sqrt{f}-\frac{bc_1-2c_2}{4-b^2}\right)}{\sigma_\beta}$.

Thus,

$$\sqrt{f} \leq \frac{\sigma_\beta z_\delta}{2+b} + \frac{1-r}{2+b} + \frac{bc_1-2c_2}{4-b^2}. \tag{A4}$$

Obviously, $\dfrac{\sigma_\beta z_\delta}{2+b}+\dfrac{1-r}{2+b}+\dfrac{bc_1-2c_2}{4-b^2}\geq 0$, from which the upper bound of r is obtained,

$$\overset{=a}{r} \equiv \sigma_\beta z_\delta + \frac{2(1-c_2)-b(1-c_1)}{2-b}. \tag{A5}$$

From the obvious condition $\overset{=a}{r} \geq 0$, the lower bound of z_δ is obtained,

$$z_\delta^a = \frac{-2+\mathrm{b}-bc_1+2c_2}{\sigma_\beta(2-\mathrm{b})}. \tag{A6}$$

$$\text{From (A4)}, f^{a*} = \left(\frac{\sigma_\beta z_\delta}{2+b}+\frac{1-r}{2+b}+\frac{bc_1-2c_2}{4-b^2}\right)^2. \tag{A7}$$

After replacing f with f^{a*}, the profit function for the port investor $\max\limits_{r,f} E_\beta(Z)$ can be rewritten as

$$\max_r E_\beta(Z) = 2\left(\frac{\sigma_\beta z_\delta}{2+b}\right)^2 + 2\left(\frac{1-r}{2+b}+\frac{bc_1-2c_2}{4-b^2}\right)^2 + r\left(\frac{1-r}{2+b}+\frac{bc_2-2c_1}{4-b^2}\right) + \left(r+\frac{4\sigma_\beta z_\delta}{2+b}\right)\left(\frac{1-r}{2+b}+\frac{bc_1-2c_2}{4-b^2}\right).$$

Note that $\dfrac{\partial^2 E_\beta(Z)}{\partial r^2} = \dfrac{-4(1+b)}{(2+b)^2} < 0$. Thus, $E_\beta(Z)$ is a concave function. By solving the first order condition for the optimal solution, $\dfrac{\partial E_\beta(Z)}{\partial r} = 0$, the optimal r^* can be obtained as follows:

$$r^{a*} = \frac{-\sigma_\beta z_\delta}{1+b} + \left(\frac{(4+b^2)c_2-(4+4b-b^2)c_1+4b-2b^2)}{4(1+b)(2-b)}\right). \tag{A8}$$

Appendix 1 proved that $\dfrac{(4+b^2)c_2-(4+4b-b^2)c_1+4b-2b^2}{4(1+b)(2-b)} > 0$. From the assumption that $\delta \leq 0.5$, $r^{a*} > 0$.

Thus, together with (A6), A(7), and (A8), the conclusion holds. ∎

Appendix 6: Finding optimal values of r and f for problem (P-P)

Considering $\pi_2^{p*}(f, r) < \pi_1^{p*}(f, r)$ for all the values of α, β, f, and r, and $\pi_i^{p*}(f, r) = (1+\beta+\alpha-q_2^{p*}-bq_1^{p*}-c_2-r)q_2^{p*}-f$, the constraint, $\mathrm{P}\{\pi_i^{p*}(f, r)\geq 0\}\geq 1-\delta$, may be rewritten as follows:

$$P\{(1+\beta+\alpha-q_2^{p*}-bq_1^{p*}-c_2-r)\, q_2^{p*}-f\geq 0\}\geq 1-\delta. \tag{A9}$$

Note that for a given value of α, β, and r, $Z = 2f + r(q_1^{p*}) + r(q_2^{p*})$ increases with increasing f. Thus, f^{p*} is the maximum f satisfying (A9). The constraint, $\mathrm{P}\{(1+\beta+\alpha-q_2^{p*}-bq_1^{p*}-c_2-r)\, q_2^{p*}-f\geq 0\}\geq 1-\delta$, is equivalent to the constraint, $\mathrm{P}\{(1+\beta+\alpha-q_2^{p*}-bq_1^{p*}-c_2-r)\, q_2^{p*}-f\leq 0\} < \delta$. The inequality, $(1+\beta+\alpha-q_2^{p*}-bq_1^{p*}-c_2-r)\, q_2^{p*}-f\leq 0$, is the same as $\beta_1\leq \beta \leq \beta_2$, where $\beta_1 = \dfrac{-w_2-\sqrt{w_2^2-4w_1w_3}}{2w_1}$, $\beta_2 = \dfrac{-w_2+\sqrt{w_2^2-4w_1w_3}}{2w_1}$, $w_1 = \left(\dfrac{1}{b+2}\right)^2$, $w_2 = \dfrac{1}{b+2}\left(\dfrac{-b(1-r)}{b+2}+1+\alpha+\dfrac{\mathrm{b}(bc_1-2c_2)}{4-b^2}-c_2-r\right)$, and $w_3 = \left(\dfrac{1-r}{b+2}+\dfrac{bc_1-2c_2}{4-b^2}\right)\left(1+\alpha-\dfrac{(1-r)(b+1)}{b+2}-\dfrac{(1-b)(bc_1-2c_2)}{4-b^2}-c_2-r\right)-f$. Thus, the constraint may be expressed as follows:

$$\int_{-\infty}^{\infty}\int_{\beta_1}^{\beta_2}\left\{\frac{1}{\sigma_\alpha\sqrt{2\pi}}\exp\left(-\frac{x^2}{2\sigma_\alpha^2}\right)\right\}\left\{\frac{1}{\sigma_\beta\sqrt{2\pi}}\exp\left(-\frac{y^2}{2\sigma_\beta^2}\right)\right\}dydx \leq \delta. \tag{A10}$$

For given values of r and f, it can be easily checked whether the inequality (A10) holds or not. Thus, for a given value of r, by searching the maximum value of f which satisfies (A10), we can find $f^{p*}(r)$.

Thus, the objective function of (P-P) can be rewritten as follows: $\max\limits_r E_{(\alpha,\beta)}(Z) = 2f^{p*}(r) + r(q_1^{d*}+q_2^{d*})$, considering $E_{(\alpha,\beta)}[q_i^{p*}] = q_i^{d*}$. The value of r^{p*}, maximising $E_{(\alpha,\beta)}(Z)$, may be found by an one dimensional numerical search. ∎

Appendix 7: Normalizing parameters

Case with a deterministic cargo demand

The linear relationships between the original price and the original cargo amount of the terminals are as follows (Zhou and Kim 2019): $\bar{p}_1 = \tau - \theta\bar{q}_1 - \omega\bar{q}_2$ and $\bar{p}_2 = \tau - \theta\bar{q}_2 - \omega\bar{q}_1$. This paper introduces a new notation, φ, which is equal to τ/θ. Then, the above equations can rewritten as $\bar{p}_1 = \tau - \theta\varphi\frac{\bar{q}_1}{\varphi} - \omega\varphi\frac{\bar{q}_2}{\varphi}$ and $\bar{p}_2 = \tau - \theta\varphi\frac{\bar{q}_2}{\varphi} - \omega\varphi\frac{\bar{q}_1}{\varphi}$. From $\bar{p}_1 > 0$ and $\bar{p}_2 > 0$, it follows that $\tau > \theta\bar{q}_1$ and $\tau > \theta\bar{q}_2$, respectively. Thus, $\varphi > \bar{q}_2$ and $\varphi > \bar{q}_1$. After replacing τ with $\theta\varphi$ and dividing by $\theta\varphi$, the two equations are changed to $\frac{\bar{p}_1}{\theta\varphi} = 1 - \frac{\bar{q}_1}{\varphi} - \frac{\omega}{\theta}\frac{\bar{q}_2}{\varphi}$ and $\frac{\bar{p}_2}{\theta\varphi} = 1 - \frac{\bar{q}_2}{\varphi} - \frac{\omega}{\theta}\frac{\bar{q}_1}{\varphi}$, respectively.

From $\theta \geq \omega$, $\varphi > \bar{q}_2$ and $\varphi > \bar{q}_1$, the inequalities $0 < \frac{\omega}{\theta} \leq 1$, $0 < \frac{\bar{q}_1}{\varphi} < 1$ and $0 < \frac{\bar{q}_2}{\varphi} < 1$ follow. Let $q_1 = \frac{\bar{q}_1}{\varphi}$, $q_2 = \frac{\bar{q}_2}{\varphi}$, $p_1 = \frac{\bar{p}_1}{\theta\varphi}$, $p_2 = \frac{\bar{p}_2}{\theta\varphi}$, and $b = \frac{\omega}{\theta}$. Finally, the normalised linear demand models are obtained as follows: $p_1 = 1 - q_1 - bq_2$ and $p_1 = 1 - q_1 - bq_2$.

About the original profit function for container terminal operators, $\bar{\pi}_1 = \bar{p}_1\bar{q}_1 - (\bar{c}_1 + \bar{r})\bar{q}_1 - \bar{f}$ and $\bar{\pi}_2 = \bar{p}_2\bar{q}_2 - (\bar{c}_2 + \bar{r})\bar{q}_2 - \bar{f}$, a similar normalisation may be done as follows. The above two equations are converted to the following new equations using $q_1 = \frac{\bar{q}_1}{\varphi}$, $q_2 = \frac{\bar{q}_2}{\varphi}$, $p_1 = \frac{\bar{p}_1}{\theta\varphi}$, and $p_2 = \frac{\bar{p}_2}{\theta\varphi}$. $\bar{\pi}_1 = (p_1\theta\varphi - \bar{c}_1 - \bar{r})\varphi q_1 - \bar{f}$ and $\bar{\pi}_2 = (p_2\theta\varphi - \bar{c}_2 - \bar{r})\varphi q_2 - \bar{f}$.

Dividing by $\theta\varphi^2$, the following equations are derived:

$$\frac{\bar{\pi}_1}{\theta\varphi^2} = \left(p_1 - \frac{\bar{c}_1}{\theta\varphi} - \frac{\bar{r}}{\theta\varphi}\right)q_1 - \frac{\bar{f}}{\theta\varphi^2} \text{ and } \frac{\bar{\pi}_2}{\theta\varphi^2} = \left(p_2 - \frac{\bar{c}_2}{\theta\varphi} - \frac{\bar{r}}{\theta\varphi}\right)q_2 - \frac{\bar{f}}{\theta\varphi^2}. \tag{A11}$$

Finally, the normalised c_1, c_2, f and r are expressed as follows:

$$c_1 = \frac{\bar{c}_1}{\theta\varphi}, c_2 = \frac{\bar{c}_2}{\theta\varphi}, r = \frac{\bar{r}}{\theta\varphi}, f = \frac{\bar{f}}{\theta\varphi^2}, \pi_1 = \frac{\bar{\pi}_1}{\theta\varphi^2}, \text{ and } \pi_2 = \frac{\bar{\pi}_2}{\theta\varphi^2}. \tag{A12}$$

Case with uncertain cargo demand

The linear relationships between the original price and the original cargo amount of the terminals are as follows: $\bar{p}_1 = A - \theta\bar{q}_1 - \omega\bar{q}_2$ and $\bar{p}_2 = A - \theta\bar{q}_2 - \omega\bar{q}_1$. Note that A is a random variable following $N(\mu, \rho^2)$. As in the case with a deterministic demand, let $\varphi = E(A)/\theta$. The above equations can then rewritten as $\bar{p}_1 = A - \theta\varphi\frac{\bar{q}_1}{\varphi} - \omega\varphi\frac{\bar{q}_2}{\varphi}$ and $\bar{p}_2 = A - \theta\varphi\frac{\bar{q}_2}{\varphi} - \omega\varphi\frac{\bar{q}_1}{\varphi}$. From $\bar{p}_1 > 0$ and $\bar{p}_2 > 0$, it follows that the range of A satisfying $A > \theta\bar{q}_1$ and $A > \theta\bar{q}_2$ is valid, which implies that $E(A) > \theta\bar{q}_1$ and $E(A) > \theta\bar{q}_2$. Thus, $\varphi > \bar{q}_2$ and $\varphi > \bar{q}_1$. Define a random variable B to be $A - E(A)$. Then, $A = B + E(A)$.

After replacing E(A) with $\theta\varphi$ and dividing by $\theta\varphi$, the two equations are changed to $\frac{\bar{p}_1}{\theta\varphi} = 1 + \frac{B}{\theta\varphi} - \frac{\bar{q}_1}{\varphi} - \frac{\omega}{\theta}\frac{\bar{q}_2}{\varphi}$ and $\frac{\bar{p}_2}{\theta\varphi} = 1 + \frac{B}{\theta\varphi} - \frac{\bar{q}_2}{\varphi} - \frac{\omega}{\theta}\frac{\bar{q}_1}{\varphi}$, respectively.

From $\theta \geq \omega$, $\varphi > \bar{q}_2$ and $\varphi > \bar{q}_1$, the inequalities $0 < \frac{\omega}{\theta} \leq 1$, $0 < \frac{\bar{q}_1}{\varphi} < 1$ and $0 < \frac{\bar{q}_2}{\varphi} < 1$ follow. Let $q_1 = \frac{\bar{q}_1}{\varphi}$, $q_2 = \frac{\bar{q}_2}{\varphi}$, $p_1 = \frac{\bar{p}_1}{\theta\varphi}$, $p_2 = \frac{\bar{p}_2}{\theta\varphi}$, $b = \frac{\omega}{\theta}$, and $\beta = \frac{B}{\theta\varphi}$. Finally, the normalised linear demand models are obtained as follows: $p_1 = 1 + \beta - q_1 - bq_2$ and $p_1 = 1 + \beta - q_1 - bq_2$. Note that β follows $N(0, \sigma_\beta^2)$, where $\sigma_\beta = \frac{\rho}{\theta\varphi}$.