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Hybrid train formation plan integrating one-block and two-block trains from a technical station under wagon demand fluctuation

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ABSTRACT

A train formation plan under wagon demand fluctuation (TFP&WDF) is provided in this article. A one-block train formation model (OBTFM) for the TFP&WDF problem is formulated with the goal of minimizing the consumption of wagon–hours. For solving the OBTFM model with three probability constraints, a procedure for transforming probability constraints into deterministic expressions and an improved genetic algorithm are presented. Four different types of two-block train are proposed to acquire a more efficient train formation plan. In terms of wagon–hours savings achieved by replacing one-block trains with four types of two-block train, a hybrid train formation model (HTFM) integrating one-block and two-block trains for TFP&WDF is developed. An heuristic is given based on covering and joint priority order for solving the proposed HTFM. Finally, a case study is presented to test the two models and the proposed approach. Compared to the one-block train formation plan, the hybrid train formation plan reduces wagon–hours consumption by 3.8%.

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Demand uncertainty; genetic algorithm; train formation

1. Introduction

The railway is an essential national logistics facility for transporting cargo (Li *et al.* 2024). The complicated and changeable market environment makes trains an essential support for supply chains. The complex situation leads directly to the fluctuation in demand for rail wagons (Habiballahi, Tamannaei, and Falsafain 2022; Zhang *et al.* 2021). The optimization of a train formation plan under wagon demand fluctuation (TFP&WDF) is presented.

Developing the best train formation plan is the foundation for railway operations. The train formation plan determines which pairs of technical stations should have train services and what types of train service should be organized. The 'technical station', which is a railway facility focused on railway traffic management and operations, in the China railway system is a special station providing a train classification service. The technical station is usually located in large transport hub cities and is equipped with an expensive marshalling yard infrastructure composed of an arrival yard, a classification bowl with a sophisticated braking system, and a departure yard. The technical station collects the wagon flow from their radiation area to form the train servicing the hub-and-spoke network. In the technical station, wagon flows with similar destinations are consolidated into a train and delivered to the destination. Wagon flows between pairs of specialized stations are usually carried out by two types of train, *i.e.* one-block trains and two-block trains from the technical station.

Some wagons with the same origin and destination (O-D) are grouped to form a block. The research reported in this article defines the block as an O-D pair. A one-block train carries one block

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and is classified as such until it reaches the destination of that block. When the one-block train arrives at the destination, the whole one-block train will break up. A direct train service carries a block from the block's origin to its destination.

A two-block train carries two blocks, composed of a long block and a short block. The long block is destined for the train's destination. However, the short block that originated from and is destined for an intermediate technical station located on its itinerary. When the two-block train arrives at the intermediate technical station, a block-exchange operation needs to be done instead of the whole train break-up. The wagon group forming a short block at the origin of the train will be replaced by a wagon group forming a short block at the intermediate technical station of the train. The former is defined as the detaching wagon group. The latter is defined as the supplement wagon group. The detaching wagon group is dropped from the two-block train, and the supplement wagon group is attached to the two-block train. Here, the intermediate technical station is defined as the block-exchange station.

Because the organization of a one-block train is simpler to implement, most freight trains in the Chinese railway system are operated as one-block trains. However, a two-block train has the advantages of reducing the train accumulation time at the origin station and simplifying the reclassification operation at the intermediate station. So, instead of selecting one of them, working out a hybrid train formation plan integrating one-block and two-block trains can improve the efficiency of freight train organization to a greater extent.

This study aims to develop the formulation and solution of a train formation plan under wagon demand fluctuation. Its core task is to work out a hybrid train formation plan, integrating one-block and two-block trains to minimize the total wagon–hours consumption through all technical stations.

The remainder of this article is organized as follows. Section 2 introduces a literature review. Section 3 presents the problem statements and mathematical formulations. Section 4 shows the proposed solution methods. A case study is shown in Section 5. Finally, conclusions are given.

2. Literature review

This article reviews the relevant literature about train formation plan organization and introduces it from the perspective of the following three groups: formation plans of one-block trains; integrated formation plans of one-block and two-block trains; and train formation plans under wagon demand fluctuation.

Many investigations have discussed one-block train formation plans using mathematical model methodology and heuristic algorithms from different perspectives. Newton, Barnhart, and Vance (1998) transformed the problem of freight train formation plans into a problem of service network design and established a linear integer programming model. The hybrid algorithm was then used to obtain an optimal scheme. Ahuja, Jha, and Liu (2007) developed a mathematical model to minimize the cost of transporting shipments, and an heuristic algorithm was proposed to optimize train formation plans. Bodin *et al.* (1980) established an arc-based mixed integer programming model considering some capacity constraints to solve the blocking problem. Yaghini, Nikoo, and Reza Ahadi (2014) aimed to maximize line utilization capacity and discussed the comprehensive optimization problem of section trains and technical trains. Lin, Tian, and Wang (2011) proposed a bi-level model to optimize the one-block train formation plan based on the remote reclassification rule. An heuristic solution approach based on simulated annealing was developed to solve the mathematical model. Mu and Dessouky (2011) investigated the train formation problem of the US railway freight transportation system with the objective of minimizing the total time consumption. Two mathematical models based on a fixed path and a flexible path were proposed, and an heuristic algorithm was developed to obtain a solution. Yaghini, Foroughi, and Nadjari (2011) established a metaheuristic model to optimize train formation while minimizing the total cost. Then, an algorithm based on ant colony optimization was presented to solve the problem. Lin *et al.* (2021) aimed at minimizing wagon–hours consumption and established a comprehensive optimization model of wagon flow routing optimization and a one-block train formation plan. A simulated annealing algorithm was developed to obtain the optimal solution. Park and Kim (2012) intended to minimize the total cost of reclassification and train operation, and proposed a model for synthetically optimizing the wagon flow routing and train formation plan. Fang, Wei, and Yang (2021) constructed a two-stage linear programming model and solution method for the integrated optimization of wagon flow routing and a one-block train formation plan at a technical station.

Some literature has been addressed from the viewpoint of optimization models and solution methods for the integrated optimization of one- and two-block train formation plans. Xiao, Lin, and Wang (2018) investigated the formation problem of one- and two-block trains in railway freight transportation to minimize total wagon–hours consumption. A hybrid algorithm consisting of a genetic algorithm and tabu search was adopted to solve the one-block train formation plan first, and then some one-block trains were converted to two-block trains. Chen *et al.* (2011a, 2011b) analysed the accumulation parameter and applicable operation conditions of two-bock trains with a fixed weight under uncertainty. Kozachenko, Bobrovskiy, and Gera (2021) investigated the train formation problem by using a two-block train and presented a mathematical statement choosing the optimal order. Liang and Lin (2006) analysed the characteristics and organization process of a two-block train, then set up a binary programming model to optimize the two-block train formation plan of the technical station. Xiao and Lin (2016) established the comprehensive optimization model of train formulation using both one- and two-block trains while minimizing the total time and developed an heuristic algorithm based on ant colony evolution to solve the formation problem.

Only a few works in the literature on train formation investigate the impact of wagon flow fluctuation. Shafia, Sadjadi, and Jamili (2010) investigated the problem of wagon flow routing and train formation plans with the wagon flow data subject to uncertainty. A comprehensive optimization model and an heuristic approach were proposed to determine the optimal solution. Yu-song, Zuo-an, and Xiao-yin (2017) established a comprehensive optimization model of wagon flow routing and a train formation plan for a one-block train under the scenario of wagon flow fluctuation. A novel solution method based on the theory of chance-constrained programming was proposed to solve the problem. Tarhini and Bish (2016) discussed the problem of network routing and wagon flow control under demand uncertainty. Two models using deterministic parameters and an heuristic approach were proposed. Niu (2003) developed a probability model and a hybrid genetic algorithm to optimize the task assignment of classification yards with wagon demand fluctuation. Li *et al.* (2023) studied a train formation plan based on a technical station under fluctuation of demand for railcars. Lordieck, Nold, and Corman (2024) studied the capacity occupation of planned and realized railway operations. Their article proposed a critical path approach to solve it. Ma *et al.* (2024) studied the daily freight train scheduling and dynamic railcar routing problems. Their article proposed two mathematical models to solve the problems studied.

The present article studies a hybrid train formation plan integrating one-block and two-block trains from technical stations under wagon demand fluctuation. The research reported in this article systematically analyses one-block trains and four different types of two-block train, including covering two-block train with a limited proportion (Covering TBT&LP), covering two-block train with an unlimited proportion (CoveringTBT&UP), joint two-block train with a limited proportion (Joint TBT&LP) and joint two-block train with an unlimited proportion (Joint TBT&UP). Wagon-hours saving is calculated by replacing a one-block train with one of the four different types of two-block train. The research reported in this article establishes a one-block train formation model to minimize wagon–hours consumption induced by consolidating wagons to form a train in the origin technical station and classifying the train at a middle technical station. Additionally, a hybrid train formation model integrating a one-block and two-block train is developed to maximize the wagon–hours savings achieved by replacing a one-block train with a two-block train. This article proposes an approach for transforming probability constraints into deterministic constraints. Additionally, an improved genetic algorithm is developed for solving the one-block train formation model, and an heuristic based on covering and joint priority order is presented for solving the hybrid train formation model.

Finally, this article tests the proposed model and approach on a railway network. The results show that the hybrid train formation plan obtained by solving the hybrid train formation model (HTFM) model has less wagon–hour consumption and provides more efficient organizational work than the one-block train formation plan.

3. Problem statement and mathematical formulation

In this section, this article first introduces a one-block train formation plan, and four different types of two-block train formation plan are systematically analysed. Moreover, wagon–hours savings generated by replacing one-block trains with four different types of two-block train are systematically investigated. Finally, this article proposes two detailed mathematical formulations for the TFP&WDF problem.

3.1. Problem definition and notation

The notation used in this article is given in the appendix (see Appendix. Notations).

3.2. Train formation plans

(1) One-block train formation plan

The one-block train formation plan aims to decide which pairs of technical stations should receive direct train services and which wagons should be picked up by each block. The research reported in this article takes the one-block train as an example. As shown in Figure 1, three technical stations in a straight railroad are denoted as i, k and j. Three blocks are built and denoted as blocks $\{i \rightarrow j\}$, $\{i \rightarrow k\}$ and $\{k \rightarrow j\}$. Block $\{i \rightarrow j\}$ is the long block. Both blocks $\{i \rightarrow k\}$ and $\{k \rightarrow j\}$ are short blocks. To distinguish the two short blocks, this article defines them, respectively, as the '1st short block' and the '2nd short block'. Three wagon flows, g_{ij}, g_{ik} and g_{kj}, exist in the straight railroad.

The research reported in this article can combine the three wagon flows to organize two types of one-block train formation plan as follows.

(i) *One-block train formation plan covering one long block and two short blocks.*
Three one-block trains independently carry wagon flows $\{g_{ij}\}$, $\{g_{ik}\}$ and $\{g_{kj}\}$ from the origin to the destination. The one-block train $\{g_{ij}\}$ serving long block $\{i \rightarrow j\}$ carries wagon flow g_{ij} independently from technical station i and breaks up after arriving at the destination technical station j. The one-block train $\{g_{ik}\}$ serving the 1st short block $\{i \rightarrow k\}$ carries wagon flow g_{ik} independently from technical station i and breaks up after arriving at the destination technical station k. The one-block train $\{g_{kj}\}$ serving the 2nd short block $\{k \rightarrow j\}$ carries wagon flow g_{kj} independently from the technical station k and breaks up after arriving at the destination technical station j.

For this type of one-block train formation plan, the long block served by one-block train $\{g_{ij}\}$ fully covers the two short blocks served by one-block trains $\{g_{ik}\}$ and $\{g_{kj}\}$.

(ii) *One-block train formation plan joining two short blocks.*
Two one-block trains carry the hybrid wagon flows $\{g_{ik} + g_{ij}\}$ and $\{g_{kj} + g_{ij}\}$, respectively. The one-block train $\{g_{ik} + g_{ij}\}$ serving the 1st short block $\{i \rightarrow k\}$ carries hybrid wagon flow $\{g_{ik} + g_{ij}\}$ from technical station i and breaks up after arriving at the destination technical station k. The other one-block train $\{g_{kj} + g_{ij}\}$ serving the 2nd short block $\{k \rightarrow j\}$ carries hybrid wagon flow $\{g_{kj} + g_{ij}\}$ from technical station k and breaks up after arriving at the destination technical station j.

For this type of one-block train formation plan, the first short block served by one-block train $\{g_{ik} + g_{ij}\}$ is adjacent to the 2nd short block served by one-block train $\{g_{kj} + g_{ij}\}$.

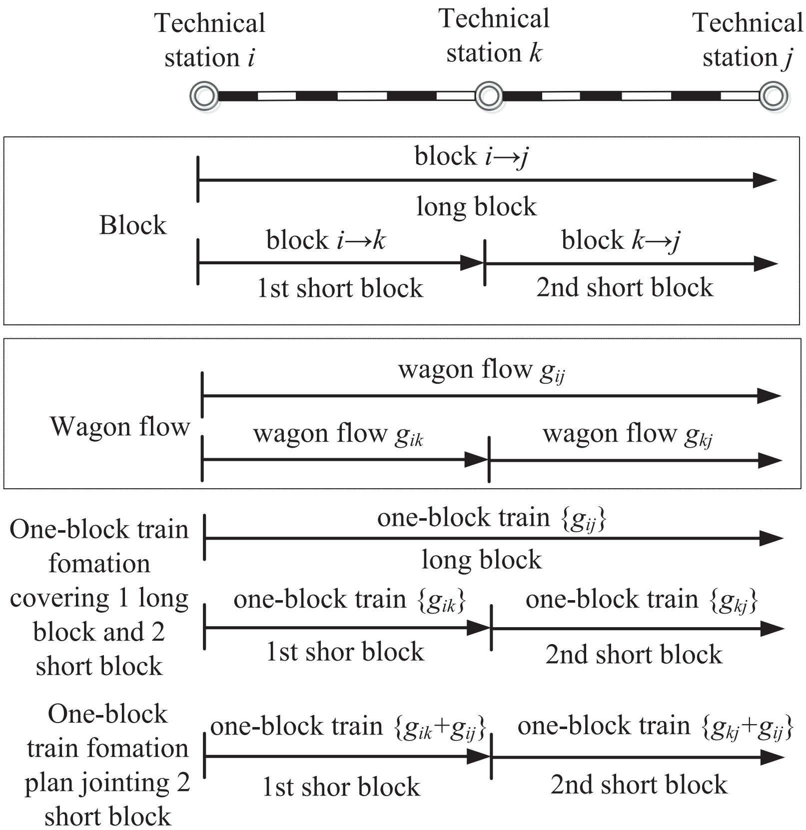

Figure 1. One-block train formation plan.

(2) *Two-block train formation plan*

Four types of two-block train formation plan can be obtained by combining the above-mentioned two types of one-block train formation plan, i.e. Covering TBT&LP, covering TBT&UP, joint TBT&LP and joint TBT&UP.

(i) *Formation plan based on covering two-block train with a limited proportion.*

The formation plan based on covering a two-block train with a limited proportion is illustrated in Figure 2. There are three one-block trains serving one long block and two short blocks, *i.e.* train $\{g_{ij}\}$ serving long block $\{i \rightarrow j\}$, train $\{g_{ik}\}$ serving the 1st short block $\{i \rightarrow k\}$ and train $\{g_{kj}\}$ serving the 2nd short block $\{k \rightarrow j\}$. Additionally, the long block served by one-block train $\{g_{ij}\}$ fully covers the two short blocks served by one-block trains $\{g_{ik}\}$ and $\{g_{kj}\}$. Technical station k is a block-exchange station. Here, the size of the 1st short-block wagon flow g_{ik} that originated at technical station i is more than the size of the 2nd short-block wagon flow g_{kj} that originated at block-exchange station k.

Three types of one-block train can be integrated to form a two-block train and an additional one-block train. The two-block train carries the long block wagon flows and the 1st short-block wagon flow at technical station i. Because the 2nd short-block wagon flow g_{kj} that originated at block-exchange station k is insufficient, the proportion of the 1st short-block wagon g_{ik} that originated at technical station i must be restricted strictly. Here, "proportion" refers to the upper limit of the extractable 1st short block wagon flow (g_{ik}) in relation to the long block wagon flow (g_{ij}), which is determined by the available amount of 2nd short block wagon flow (g_{kj}). This upper limit is defined by the ratio g_{kj}/g_{ij}. If the proportion is too high, the dropping amount of the 1st short-block wagon flow will be more than

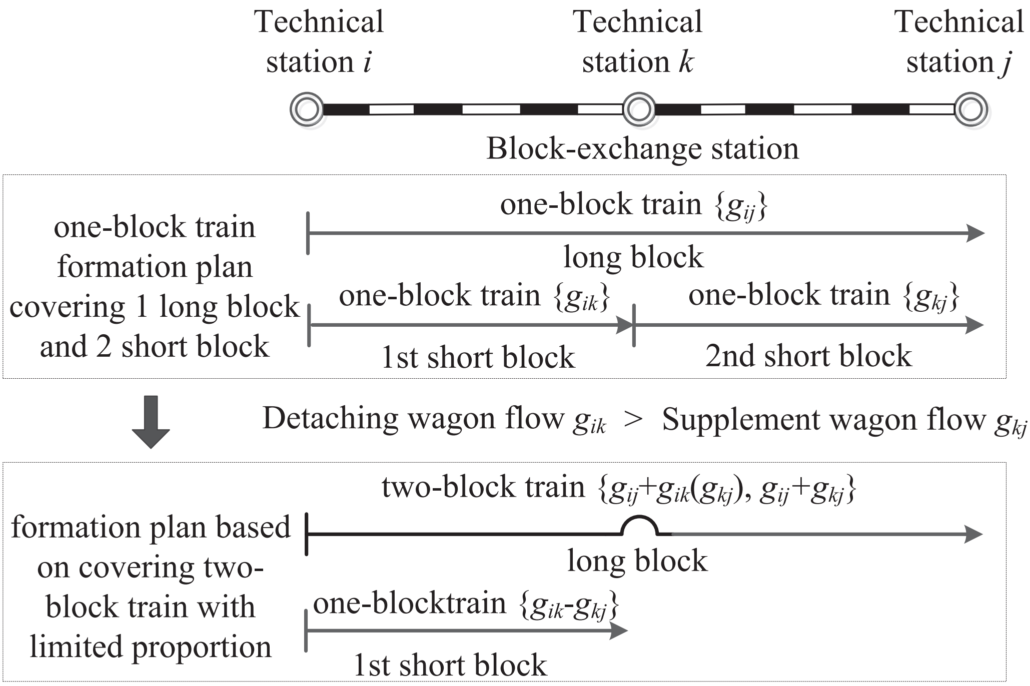

Figure 2. Formation plan based on covering a two-block train with a limited proportion.

the supplement amount of the 2nd short-block wagon flow after arriving at block-exchange station k. So the number of wagons forming the train cannot satisfy the specified train size, and then the train will have to be broken up at the block-exchange station k. Based on the above reasons, the proportion of the 1st short-block wagon flow g_{ik} and long block wagon flow g_{ij} forming the two-block train at origin station i must be limited in value, *i.e.* g_{kj}/g_{ij} instead of g_{ik}/g_{ij}. That is to say, only a wagon flow with the size of g_{kj} needs to be taken out from the 1st short-block wagon flow g_{ik} to consolidate into the two-block train at the origin station i instead of all wagon flow g_{ik}. The extracted 1st short-block wagon flow is denoted as $g_{ik}(g_{kj})$. When the two-block train $\{g_{ij} + g_{ik}(g_{kj}), g_{ij} + g_{kj}\}$ arrives at the block-exchange station k, the 1st short-block wagon flow whose proportion in the whole two-block train is g_{kj}/g_{ij} can be replaced by the 2nd short-block wagon flow of the same amount . The remaining wagon flow $g_{ik} - g_{kj}$ will be collected together to form an additional one-block train $\{g_{ik} - g_{kj}\}$ serving the 1st short block at technical station i.

(ii) ***Formation plan based on covering a two-block train with an unlimited proportion.*** When the size of the 1st short-block wagon flow g_{ik} that originated at technical station i is less than the size of the 2nd short-block wagon flow g_{kj} that originated at block-exchange station k, three one-block trains covering one long block train and two short-block trains can be converted to the formation plan based on covering the two-block train with an unlimited proportion illustrated in Figure 3.

Similarly, the two-block train carries the long block wagon flows and the 1st short-block wagon flow at technical station i. The proportion of the 1st short-block wagon flow and long block wagon flows forming the two-block train is unlimited. Because the flow of the 2nd short-block supplement wagon that originated at block exchange station k is sufficient, the number of wagons can also arrive at the specified train size after the 1st short-block wagon is dropped. That is to say that all the 1st short-block wagon flow g_{ik} can be taken out to consolidate into the two-block train at origin station i. Meanwhile, only a wagon flow with the size of g_{ik} needs to be taken out from the 2nd short-block wagon flow g_{kj} to consolidate into a two-block train at block-exchange station k instead of all wagon

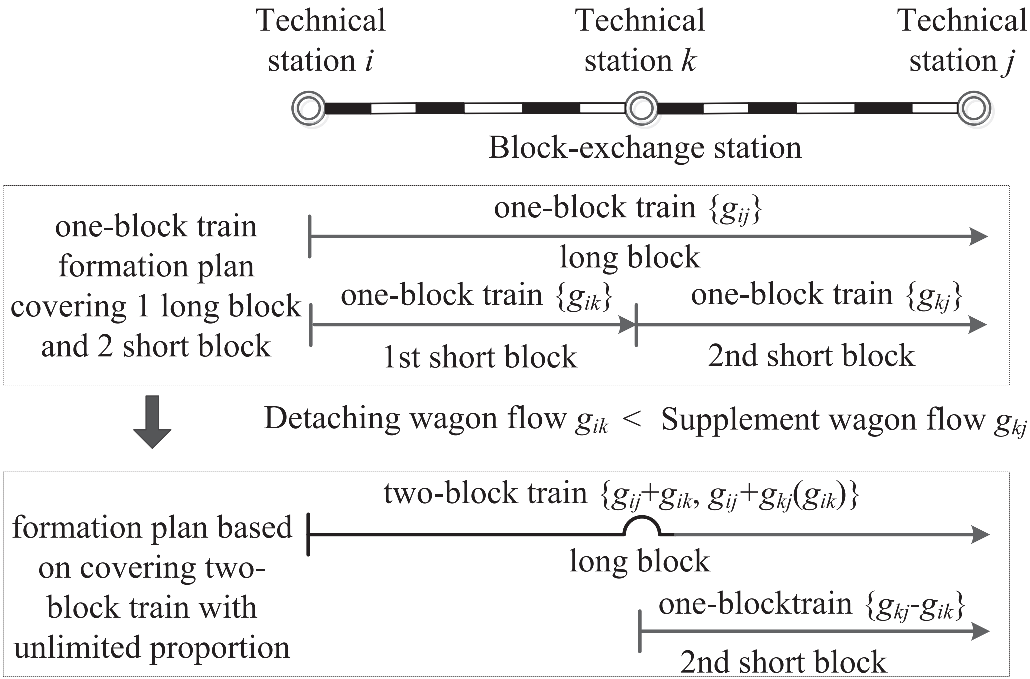

Figure 3. Formation plan based on covering a two-block train with an unlimited proportion.

flow g_{kj}. The extracted 2nd short-block wagon flow is denoted as $g_{kj}(g_{ik})$. When the two-block train $\{g_{ij} + g_{ik}, g_{ij} + g_{kj}(g_{ik})\}$ arrives at block-exchange station k, the 1st short-block wagon flow g_{ik} will be replaced by the 2nd short-block wagon flow g_{kj} of the same size . The remaining wagon flow $g_{kj} - g_{ik}$ will be collected to form an additional one-block train $\{g_{kj} - g_{ik}\}$ serving the 2nd short block at technical station k.

(iii) *Formation plan based on a joint two-block train with a limited proportion.*

A formation plan based on a joint two-block train with a limited proportion is illustrated in Figure 4. There are two one-block trains serving two adjacent short blocks, *i.e.* train $\{g_{ik} + g_{ij}\}$ serving the 1st short block $\{i \rightarrow k\}$ and $\{g_{kj} + g_{ij}\}$ serving the 2nd short block $\{k \rightarrow j\}$. Meanwhile, the 1st short block served by one-block train $\{g_{ik} + g_{ij}\}$ is adjacent to the 2nd short block served by one-block train $\{g_{kj} + g_{ij}\}$. Here, the size of the 1st short-block wagon flow g_{ik} that originated at technical station i is more than the size of the 2nd short-block wagon flow g_{kj} that originated at block-exchange station k.

The formation plan is based on a joint two-block train with a limited proportion, which is the same as the formation plan based on covering a two-block train with a limited proportion. The two-block train $\{g_{ij} + g_{ik}(g_{kj}), g_{ij} + g_{kj}\}$ serves the long block $\{i \rightarrow j\}$, and the one-block train $\{g_{ik} - g_{kj}\}$ serves the 1st short block.

(iv) *Formation plan based on a joint two-block train with an unlimited proportion.*

A formation plan based on a joint two-block train with an unlimited proportion is illustrated in Figure 5. Here, the size of the 1st short-block wagon flow g_{ik} that originated at technical station i is less than the size of the 2nd short-block wagon flow g_{kj} that originated at block-exchange station k.

The formation plan is based on a joint two-block train with an unlimited proportion, which is the same as the formation plan on covering a two-block train with an unlimited proportion. The two-block train $\{g_{ij} + g_{ik}, g_{ij} + g_{kj}(g_{ik})\}$ serves the long block $\{i \rightarrow j\}$, and the one-block train $\{g_{kj} - g_{ik}\}$ serves the 2nd short block.

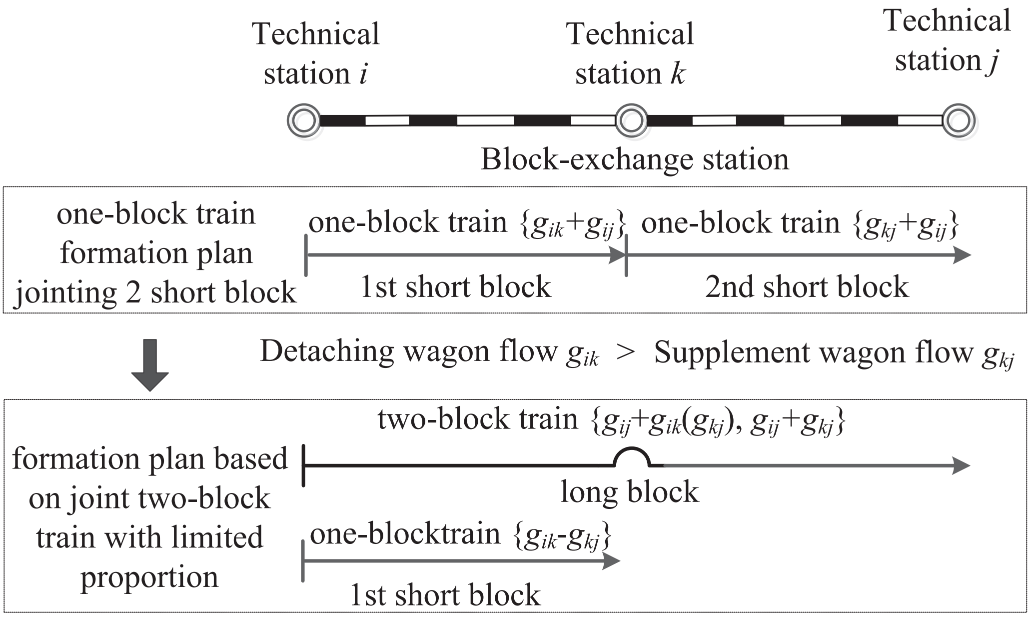

Figure 4. Formation plan based on a joint two-block train with a limited proportion.

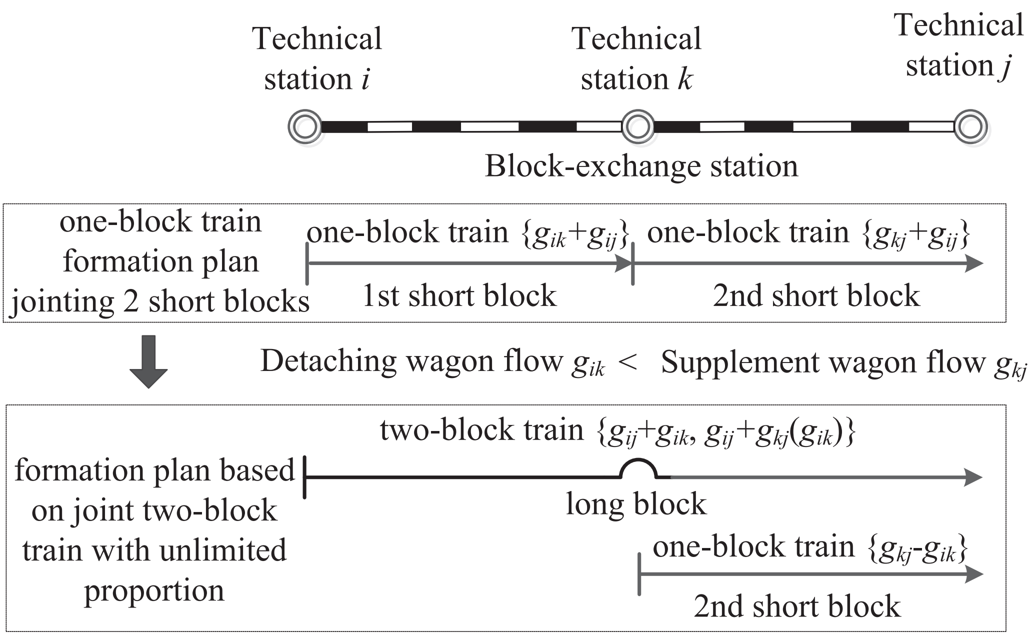

Figure 5. Formation plan based on a joint two-block train with an unlimited proportion.

3.3. Wagon–hours savings of a two-block train

In this section, the wagon–hours consumption savings generated by replacing one-block trains with two-block trains are analysed from two aspects: the departure station and the block-exchange station.

(1) *Wagon–hours at the departure station*

Wagon–hours consumption at the departure station include accumulated wagon–hours, classification wagon–hours and waiting wagon–hours.

(i) *Accumulated wagon–hours.*

For a one-block train formation plan covering one long block and two short blocks, the total accumulated wagon–hours of two one-block trains at the departure station are $2c_i m$. For a one-block train formation plan joining two short blocks, the total accumulated wagon–hours of a single one-block train at the departure station are $c_i m$.

For covering TBT&LP and joint TBT&LP, the proportion of the 1st short-block wagon flow and long block wagon flow in the same two-block train is the fixed value g_{kj}/g_{ij}. Both of their total accumulated wagon–hours are $c_i m g_{ij}(g_{ik}+g_{kj})/[g_{ik}(g_{ij}+g_{kj})]$.

For covering TBT&UP and joint TBT&UP, the proportion of the 1st short-block wagon flow and long block wagon flow in the same two-block train is variable instead of a fixed value. Both of their total accumulated wagon–hours are $c_i m$.

(ii) *Classification wagon–hours.*

For the above mentioned two types of one-block train, both of their total classification wagon–hours are $t_a(g_{ij}+g_{ik})$. For covering TBT&LP and joint TBT&LP, the proportion is the fixed value g_{kj}/g_{ij}. Both classification wagon–hours are $t_i^b(g_{ij}+g_{kj})+t_a(g_{ik}-g_{kj})$.

For covering TBT&UP and joint TBT&UP, the proportion is variable instead of a fixed value. Both of their classification wagon–hours are $t_i^b(g_{ij}+g_{ik})$.

(iii) *Waiting wagon–hours.*

Because the one-block train will be organized as long as the number of wagons grouped to form a one-block train is full, the waiting wagon–hours is zero.

For covering TBT&LP and joint TBT&LP, the proportion is the fixed value g_{kj}/g_{ij}. If the number of wagons grouped to form a two-block train is full, but a proportion does not arrive, the two-block train must continue to wait at the departure station. Both of their waiting wagon–hours are $c_i m g_{ij} g_{kj}/[g_{ik}(g_{ij}+g_{kj})]$.

Covering TBT&UP and joint TBT&UP, the proportion is not limited to taking any value. Both of their waiting wagon–hours are zero.

(2) *Wagon–hours at the block-exchange station*

Wagon–hours at the block-exchange station include accumulated wagon–hours, waiting wagon–hours, reclassification wagon–hours and the wagon–hours delay of the long block wagon.

(i) *Accumulated wagon–hours.*

For the above-mentioned two types of one-block train, both of their total accumulated wagon hours are $c_i m$.

For covering TBT&LP and joint TBT&LP, the proportion is the fixed value g_{kj}/g_{ij}, and both of their total accumulated wagon–hours are $c_k m g_{kj}/(g_{ij}+g_{kj})$.

For covering TBT&UP and joint TBT&UP, the proportion is variable instead of a fixed value. Both of their total accumulated wagon–hours are $c_k m(g_{ij}g_{kj}+g_{ik}g_{kj}-g_{ij}g_{ik})/[g_{ik}/(g_{ij}+g_{ik})]$.

(ii) *Waiting wagon–hours.*

Waiting time is caused by the inconsistency between the arrival time of a two-block train and the accumulated time of the supplement wagon group at the block-exchange station. Time asynchrony leads to one of them waiting at the block-exchange station.

For covering TBT&LP and joint TBT&LP, the proportion is strictly restricted to the fixed value g_{kj}/g_{ij}. Both of their waiting wagon–hours are $c_k m g_{kj}/(g_{ij} + g_{kj})$.

For covering TBT&UP and joint TBT&UP, the proportion is not limited to taking any value. Both of their waiting wagon–hours are $c_k m g_{ij} g_{ik}/g_{kj}(g_{ij} + g_{ik})$.

(iii) *Reclassification wagon–hours.*
Because a one-block train will break up after arriving at the destination station, the reclassification wagon–hours are zero.

For covering TBT&LP and joint TBT&LP, the proportion is a fixed value g_{kj}/g_{ij}. Two-block trains need to drop the detaching wagon group and attach the supplement wagon group. Obviously, the reclassification time of a two-block train at the block-exchange station equals the total time consumption generated by dropping the detaching wagon group and attaching the supplement wagon group at the block-exchange station. The reclassification wagon–hours are a product of the reclassification time, and the total number of wagons is composed of a detaching wagon group and an attaching supplement wagon group. So both of their total reclassification wagon–hours are $g_{kj}(t_h^z + t_h^c)$.

For covering TBT&UP and joint TBT&UP, the proportion is variable instead of a fixed value. Both of their total reclassification wagon–hours are $g_{ik}(t_h^z + t_h^c)$.

(iv) *Wagon–hours delay of a long block wagon.*
A two-block train needs to drop the detaching wagon group and attach the supplement wagon group at the block-exchange station. Wagons serving long block wagons must wait at the block-exchange station. The delay time of a long block wagon also equals the total time consumption generated by dropping the detaching wagon group and attaching the supplement wagon group at the block-exchange station. The reclassification wagon–hours are a product of the delay time and the number of long-block wagons . The wagon–hours delay of the above mentioned four types of two-block train are $t_k^l g_{ij}$.

In summary, seven types of wagon–hours savings generated at the departure and block-exchange stations are listed in Table 1. The total wagon–hours savings of the four types of two-block train are abbreviated as $\triangle t^{cl}$, $\triangle t^{cu}$, $\triangle t^{jl}$ and $\triangle t^{jl}$.

3.4. One-block train formation model

The one-block train formation model (OBTFM) for the TFP&WDF problem is formulated as a mathematical programming model whose objective function and constraints are expressed as follows:

(OBTFM) Min G

$$\text{s.t.} \quad Pr\left\{\sum_{i\in A}\sum_{j\in A} c_i m x_{ij} + \sum_{i\in A}\sum_{j\in A}\sum_{k\in A} f_{ij}^h x_{ij}^h t_k \le G\right\} \ge \eta, \quad h \in H \tag{1}$$

$$Pr\left\{\sum_{i\in P_K}\sum_{j\in Q_K} f_{ij}^h x_{ij}^h \le b_k \lambda_k\right\} \ge \delta, \quad k \in P, \ h \in H \tag{2}$$

$$Pr\left\{\sum_{j\in A} g_{ij}^h/d_i \le \mu_i\right\} \ge \varepsilon, \quad i \in A, \ h \in H \tag{3}$$

$$f_{ij}^h = n_{ij}^h + \sum_{s\in Q_i} x_{sj}^i f_{sj}^h, \quad i,j \in A, \ h \in H \tag{4}$$

$$g_{ij}^h = f_{ij}^h x_{ij} + \sum_{e\in P_i} f_{ie}^h x_{ie}^j, \quad i,j \in A, \ h \in H \tag{5}$$

Table 1. Wagon–hours consumption savings by replacing a one-block train with a two-block train.

		Covering TBT&LP	Covering TBT&UP	Joint TBT&LP	Joint TBT&UP
Wagon–hours savings at departure station	Accumulation	$c_i m g_{kj}(g_{ij}+g_{ik})/[g_{ik}(g_{ij}+g_{kj})]$	$c_i m$	$c_i m g_{ij}(g_{ik}-g_{kj})/[g_{ik}(g_{ij}+g_{kj})]$	0
	Classification	$(t_a-t_i^b)(g_{ij}+g_{kj})$	$(t_a-t_i^b)(g_{ij}+g_{ik})$	$(t_a-t_i^b)(g_{ij}+g_{kj})$	$(t_a-t_i^b)(g_{ij}+g_{ik})$
	Waiting	$-c_i m g_{ij} g_{kj}/[g_{ik}(g_{ij}+g_{kj})]$	–	$-c_i m g_{ij} g_{kj}/[g_{ik}(g_{ij}+g_{kj})]$	–
Wagon–hours savings at block-exchange station	Accumulation	$c_k m(g_{ij}-g_{kj})/g_{ij}$	$c_k m g_{ij} g_{ik}/g_{kj}(g_{ij}+g_{ik})$	$c_k m(g_{ij}-g_{kj})/g_{ij}$	$c_k m g_{ij} g_{ik}/g_{kj}(g_{ij}+g_{ik})$
	Waiting	$-c_k m g_{kj}/(g_{ij}+g_{kj})$	$-c_k m g_{ij} g_{ik}/g_{kj}(g_{ij}+g_{ik})$	$-c_k m g_{kj}/(g_{ij}+g_{kj})$	$-c_k m g_{ij} g_{ik}/g_{kj}(g_{ij}+g_{ik})$
	Reclassification	$-g_{kj}(t_h^z+t_h^c)$	$-g_{ik}(t_h^z+t_h^c)$	$g_{ij}t_k-g_{kj}(t_h^z+t_h^c)$	$g_{ij}t_k-g_{ik}(t_h^z+t_h^c)$
	Delay of long block wagon	$t_k^l g_{ij}$	$t_k^l g_{ij}$	$t_k^l g_{ij}$	$t_k^l g_{ij}$
Total wagon–hours savings		$\triangle t^{cl}$	$\triangle t^{cu}$	$\triangle t^{jl}$	$\triangle t^{jl}$

$$x_{ij} + x_{ij}^k = 1, \quad i \in P_k, \ j \in Q_k, \ k \in A \tag{6}$$

$$x_{ij}^k \leq x_{ik}, \quad i \in P_k, \ j \in Q_k, \ k \in A \tag{7}$$

$$x_{ik}, x_{ij}^k \in \{0, 1\}, \quad i, j, k \in A. \tag{8}$$

In the OBTFM model, the constraints (1), (2) and (3) are probability expression equations, and the remaining constraints are deterministic expression equations. The three probability constraints are the key to solving the model. This article first explains the three probability constraints and then the remainder of the deterministic constraints.

(1) *Probability constraint of one-block train formation plan with high wagon–hours consumption*

The strategy of accepting inferior solutions within a maximum allowable range is given to improve the solution's performance. A new probability constraint, which represents accepting the inferior solution under a certain probability, is developed. So constraint (1) is the probability constraint of a one-block train formation plan with high wagon–hours consumption, where the formula $\sum_{i \in A} \sum_{j \in A} c_i m x_{ij} + \sum_{i \in A} \sum_{j \in A} \sum_{k \in A} f_{ij}^h x_{ij}^h t_k$ is used to calculate the wagon–hours consumption of a one-block train formation plan at the specific statistical time period h, and the parameter η represents the acceptance rate of an inferior solution with a value in the range $[0.9, 1]$. Here, the inferior solution denotes a one-block train formation plan with high wagon–hours consumption. Next, the proposal termed 'one-block train formation plan with high wagon–hours consumption' is defined.

Because the planning horizon includes $\widetilde{h}$ statistical periods, the number of one-block train formation plans that can be obtained is the same as the number of periods can be obtained. The research reported in this article uses dynamic wagon flow data to calculate the wagon–hours consumption of each one-block formation plan in the range of the planning horizon. Then, this article ranks the one-block train formation plans in ascending order of wagon–hours consumption and obtains a one-block train formation plan sequence. The one-block train formation plan whose precedence is the position behind η in the sequence is defined as a one-block train formation plan with high wagon–hours consumption. Let G be the set of one-block train formation plans with high wagon–hours consumption. Therefore, constraint (1) implies that all one-block train formation plans whose wagon–hours consumptions are in set G can be accepted. The number of acceptable solutions is $\eta\widetilde{h}$.

(2) *Probability constraint of classification capacity restriction*

The research reported in this article allows a certain proportion of solutions that violate the technical station classification capacity restriction to be accepted. Constraint (2) is the probability of classification capacity restriction, where $\sum_{i \in P_K} \sum_{j \in Q_K} f_{ij}^h x_{ij}^h \leq b_k \lambda_k$ represents the fact that the number of classified wagons in each technical station must be less than its classification capacity. The parameter δ is the rate of satisfaction of the technical station classification capacity restriction, which means the proportion of the solutions that meet the technical station classification capacity restriction and are within the range $[0.9, 1]$. Thus, constraint (2) represents the fact that the proportion of one-block train formation plans satisfying the classification capacity restriction is not less than δ in the $\widetilde{h}$ train formation plans. That is to say, the number of one-block train formation plans satisfying the classification capacity restriction is not less than $\delta\widetilde{h}$.

(3) *Probability constraint restricting the number of classification tracks*

In the same approach, this article also allows a certain proportion of solutions violating the classification tracks number restriction to be accepted. Constraint (3) is the probability constraint of classification tracks number restriction, where $\sum_{j\in A} g_{ij}^{h}/d_i \le \mu_i$ represents the fact that the number of occupied classification tracks must be less than the maximum limitation of each technical station. The parameter ε is the rate of satisfaction of the classification tracks number restriction, which means the proportion of solutions meeting the classification tracks number restriction and is within the range $[0.9, 1]$. Constraint (3) guarantees that the proportion of one-block train formation plans satisfying the classification tracks number restriction is not less than ε in the $\widetilde{h}$ train formation plans. That is, the number of one-block train formation plans satisfying the classification tracks number restriction is not less than $\varepsilon\widetilde{h}$.

(4) *Deterministic constraint*

The remainder of the constraints are all deterministic. Constraint (4) indicates that the volume of actual wagon flow equals the total volume of wagon flow originating from or being reclassified at technical station i and arriving at technical station j within any statistical period. Constraint (5) indicates that the volume of service wagon flow equals the total volume of wagon flow originating from or being reclassified at technical station i and being reclassified at or arriving at technical station j within any statistical period. Constraint (6) guarantees that each wagon flow can either be directly delivered to the destination or destined for the destination after being reclassified at more than one intermediate technical station on its itinerary. Constraint (7) represents how to decide the first technical station at which a wagon flow is reclassified on its itinerary. Constraint (8) is a binary restriction on the decision variables.

3.5. *Hybrid train formation model*

To improve the performance of the solution, this article sets up a hybrid train formation model (HTFM), integrating one-block and two-block trains for the TFP&WDF. Compared with the one-block train formation model (OBTFM), the hybrid train formation model is a deterministic model whose objective function and constraints are expressed as follows:

$$\begin{aligned}(\text{HTFM})\text{MaxZ} = \sum_{i\in A}\sum_{j\in A}\sum_{k\in A} y_{ij}^{k}[u_{ij}^{k}\left(1-v_{ij}^{k}\right)\Delta t^{rl} + u_{ij}^{k}v_{ij}^{k}\Delta t^{ru} \\ + \left(1-u_{ij}^{k}\right)\left(1-v_{ij}^{k}\right)\Delta t^{jl} \\ + \left(1-u_{ij}^{k}\right)v_{ij}^{k}\Delta t^{ju}]\end{aligned} \tag{9}$$

$$\text{s.t.}\quad y_{ij}^{k} \le u_{ij}^{k} + v_{ij}^{k},\quad \forall i,j,k\in A \tag{10}$$

$$\sum y_{ij}^{k} + \sum y_{kj}^{i} = \sum y_{ik}^{j} \le 1,\quad \forall i,j,k\in A \tag{11}$$

$$y_{ij}^{k} = x_{ij}x_{ik}x_{kj},\quad \forall i,j,k\in A \tag{12}$$

$$y_{ij}^{k} = x_{ik}x_{kj},\quad \forall i,j,k\in A \tag{13}$$

$$y_{ij}^{k}, u_{ij}^{k}, v_{ij}^{k} \in \{0,1\},\quad \forall i,j,k\in A. \tag{14}$$

The objective function (9) aims to maximize the wagon–hours consumption savings by replacing one-block trains with four types of two-block train. It includes four components. The first term is wagon–hours savings by covering TBT&LP. The second term is wagon–hours savings by covering TBT&UP. The third term is wagon–hours savings by joint TBT&LP. The last term is wagon–hours savings by joint TBT&UP. Constraint (10) guarantees that a two-block train must be one of four types, *i.e.* covering TBT&LP, covering TBT&UP, joint TBT&LP and jointTBT&UP. Constraint (11) represents that, if wagon flow is organized into two-block trains, they must be one of long block and one of short block . Constraint (12) implies that three one-block trains serving one long block and two short blocks can be organized into one covering two-block train. Constraint (13) means that two one-block trains serving two adjacent short-block trains can be organized into one joint two-block train. Constraint (14) is a binary restriction on the decision variables.

4. Methodology

In order to solve the one-block train formation model, including probability constraints, this article first develops the procedure for transforming probability constraints into deterministic expressions. The genetic algorithm is a very popular algorithm that has been used to solve many optimization problems (Gao *et al.* 2017). An improved genetic algorithm (IGA) for solving the one-block train formation model is developed. Finally, an heuristic is presented based on covering and joint priority order for solving the hybrid train formation model.

4.1. Deterministic transformation of probability constraints

In the OBTFM model, the constraints (1), (2) and (3) are formulated as inequality probabilities. They are unclear mathematical analytical expressions. If these probabilistic constraints are not treated, it will lead to difficulty in solving the model. Here, this article expands the statistical wagon flow data and then transforms the probabilistic constraints into deterministic expressions.

(1) Expanding statistical wagon flows data based on discrete uniform distributions

When the statistical wagon flow data in the statistical period is insufficient, the following numerical simulation method is used to expand it.

Step 1: The research reported in this article finds the minimum value u_1 and maximum value u_2 from wagon flow n_{ij}^h between pairs of technical stations i to j in period $\widetilde{h}$. Because discrete uniform distributions have excellent characteristics, every discrete value within the value range of a random variable has the same probability of occurrence. So this article uses this distribution to describe the wagon flow random variable and denotes it as $n_{ij}^h \sim U(u_1, u_2)$.

Step 2: The research reported in this article generates $\widetilde{r}$ random numbers in $[0, 1]$, denoted as $R = \{r \mid 0 \leq r \leq 1\}$. Let $\widetilde{r} \gg \widetilde{h}$.

Step 3: The research reported in this article uses the formula $n_{ij}^r = ru_1 + (1 - r)u_2$ to generate $\widetilde{r}$ new wagon flow data n_{ij}^r. So, the expanding data set of fluctuating wagon flows can be obtained and denoted as W^R.

(2) Finding a one-block train formation plan with the lowest high wagon–hours consumption

The acceptance rate of inferior solutions is used to find the one-block train formation plan with the lowest wagon–hours consumption from the set G by the expanding wagon flows data set.

Step 1: Because there are $\widetilde{r}$ expanding wagon flow data in set W^R, the acceptance rate of the one-block train formation plan with high wagon–hours consumption needs be recounted by the formula $\omega = [\eta\widetilde{r}]$, which denotes the minimum integer greater than $\eta\widetilde{r}$.

Step 2: The research reported in this article converts the OBTFM model into a deterministic model. Here, probability constraints (1), (2) and (3) are transformed into the deterministic equations $\sum_{i\in A}\sum_{j\in A} c_i m x_{ij} + \sum_{i\in A}\sum_{j\in A}\sum_{k\in A} f_{ij}^h x_{ij}^h t_k = G$, $\sum_{i\in P_K}\sum_{j\in Q_K} f_{ij}^h x_{ij}^h \leq b_k \lambda_k$ and $\sum_{j\in A} g_{ij}^h / d_i \leq \mu_i$. So, the reconstruction OBTFM model can be obtained.

Step 3: The $\widetilde{r}$ expanding wagon flow data in set W^R are put into the reconstruction OBTFM model in turn. The reconstruction OBTFM model based on expanding wagon flow data is solved $\widetilde{r}$ times by the improved genetic algorithm proposed in Section 4.1, and $\widetilde{r}$ solutions can be obtained.

Step 4: The $\widetilde{r}$ solutions are sorted by the objective function value in ascending order. Their objective functions are denoted as $\{Z'_1, \ldots, Z'_\omega, \ldots, Z'_{\widetilde{r}}\}$, where $\theta = \widetilde{r} - \psi + 1$. Obviously, the one-block train formation plan set with high wagon–hours consumption is $G = \{Z'_\omega, \ldots, Z'_{\widetilde{r}}\}$. The one-block train formation with the lowest wagon–hours consumption in the set G is Z'_ω.

(3) *Counting rate of satisfaction of one-block train formation plans railway network capacity*

The following procedure is used to test the rate of satisfaction of technical station classification capacity and classification track number capacity of a one-block train formation plan by the expanding wagon flow data.

Step 1: All train formation plans and all $\widetilde{r}$ expanding wagon flow data in the set W^R are placed into the inequalities $\sum_{i\in P_K}\sum_{j\in Q_K} f_{ij}^h x_{ij}^h \le b_k\lambda_k$ and $\sum_{j\in A} g_{ij}^h/d_i \le \mu_i$ for judging whether the classification capacity restriction and classification tracks number restriction are both satisfied or not. The amount of expanding wagon flow data satisfying the two capacity restrictions is recorded and denoted as $\tilde{r}'$.

Step 2: The satisfaction rate of two capacity restrictions of a one-block train formation plan can be counted by the formula $\tilde{r}'/\tilde{r}$.

4.2. *Improved genetic algorithm for OBTFM*

The genetic algorithm is a popular optimization method that has been widely used in engineering optimization problems (Fan *et al.* 2023; Gao *et al.* 2017; Liu et al. 2024). In this section, an improved genetic algorithm for solving the proposed OBTFM is developed. First, an initial one-block train formation plan set is generated. Secondly, an iterative procedure based on an improved genetic algorithm is designed to optimize the initial one-block train formation plan set. Thirdly, a filtering procedure for the one-block train formation plan set is developed to find the optimal one-block train formation plan.

Stage 1: Generating an initial one-block train formation plan set

Step 1.1: Encoding wagon flow without reclassification on its itinerary.

The wagon flow n_{ij}^h that originates at technical station i and travels to j without reclassification is expressed with variables x_{ij} and x_{ij}^k.

The value of the decision variable x_{ij} indicates whether the wagon flow may be assigned to a direct block or a sequence of blocks along its journey. If $x_{ij} = 1$, the wagon flow n_{ij}^h is assigned into a direct block $i \to j$, which means that n_{ij}^h will not be reclassified until it reaches the destination j of that block. If $x_{ij} = 0$, the wagon flow n_{ij}^h is assigned into a sequence of blocks, which will be reclassified once or more along its journey.

The value of the decision variable x_{ij}^k indicates block assignment decisions. If $x_{ij}^k = 1$, the wagon flow n_{ij}^h is assigned into a block $i \to k$, which means that technical station k is the first reclassification technical station of the wagon flow n_{ij}^h from the technical station i to j.

The wagon flow without reclassification on its journey can be well-determined by the values of x_{ij} and x_{ij}^k, as is illustrated in Figure 6.

Step 1.2: Encoding wagon flow with reclassification on its journey.

In the same approach, this article can encode the wagon flow with reclassification on its journey by the variables x_{ij} and x_{ij}^k, as displayed in Figure 7.

Step 1.3: O-D wagon flows sequence.

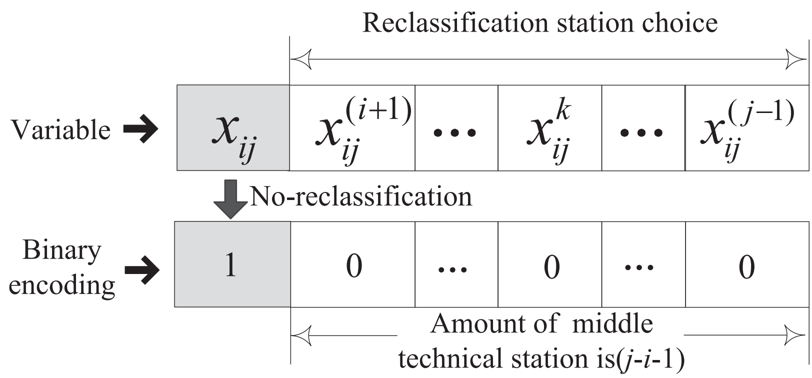

Figure 6. Encoding wagon flow without reclassification.

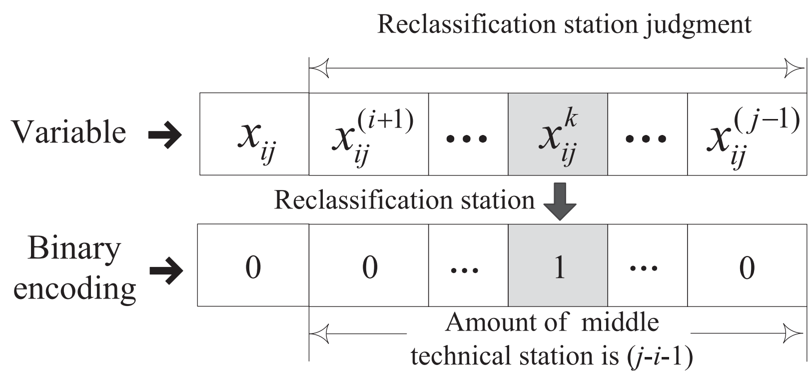

Figure 7. Encoding wagon flow with reclassification.

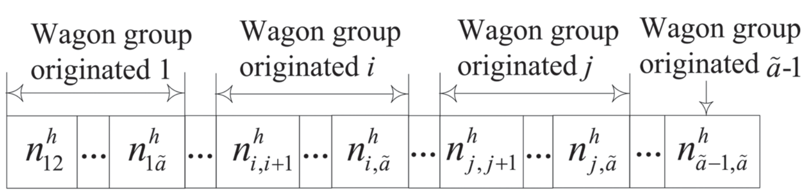

Figure 8. O-D wagon flows sequence.

An O-D pair can be generated between any two technical stations. So the number of O-D pairs with $\tilde{a}$ technical stations is $C_{\tilde{a}}^{2} = \tau$. The O-D pairs are sorted by order of departure and destination stations, as shown in Figure 8. The wagon flow group originated from technical station 1 is denoted as $\{n_{12}^{h}, L, n_{1\tilde{a}}^{h}\}$. The wagon flow group originated from technical station i is denoted as $\{n_{i,i+1}^{h}, L, n_{i\tilde{a}}^{h}\}$. The wagon flow group originated from technical station j is denoted as $\{n_{j,j+1}^{h}, L, n_{j\tilde{a}}^{h}\}$ and the wagon flow group originated from technical station $(\tilde{a} - 1)$ is denoted as $\{n_{\tilde{a}-1,\tilde{a}}^{h}\}$.

Step 1.4: O-D wagon flows coding sequence.

The wagon flow n_{ij}^{h} is selected from the O-D wagon flow sequence in turn. Then, this article randomly chooses one of the two coding schemes shown in Figures 6 and 7 to encode the selected wagon flow. When all wagon flows in the O-D wagon flow sequence shown in Figure 8 are encoded, the O-D wagon flow coding sequence is obtained.

Step 1.5: Converting O-D wagon flow coding sequences into one-block train formation plans.

Counting service wagon flows of block $i \to j$

Service wagon flows are assigned into block $i \to j$, which means the number of wagon flows that originate or reclassify at technical stations *i* and *j*, consisting of four parts.

The first one is the double-reclassified wagon flow that originates at the rear technical station of technical station *i*, reclassifies twice at technical stations *i* and *j*, and is then destined for the front technical station of technical station *j*.

The second one is the single-reclassified wagon flow that originates at the rear technical station of technical station *i*, reclassifies at technical station *i*, and then is destined for technical station *j*.

The third one is the single-reclassified wagon flow that originates at technical station *i*, reclassifies at technical station *j*, and is then destined for the front technical station of technical station *j*.

The fourth one is the original wagon flow that originates at technical station *i* and is destined for technical station *j*.

An illustration describing service wagon flows and their coding sequence of block $i \to j$ is shown in Figure 9.

Step 1.6: Initial one-block train formation plan set.

The size of the initial one-block train formation plan set is set as Ω. Steps 1.1 to Step 1.5 are repeatedly executed to generate the one-block train formation plan. Then, this paper is put into the classification capacity restriction, and classification tracks the number restriction of reconstruction OBTFM model to judge whether this article is feasible or not . The above procedure is repeated until the number of feasible one-block train formation plans arrive at Ω. This article denotes the initial one-block train formation plan set as $\Lambda(0)$.

Stage 2: Iterative procedure based on an improved genetic algorithm

Step 2.1: Setting the control parameters of the iteration procedure.

Let the initial wagon flow reclassification sequence $\Lambda(0)$ be the initial iteration population. The one-block train formation plan is made as the implementing object in the iteration procedure. The implementing object will be continually updated using the proposed approaches. The iteration population is composed of Ω implementing objects. Let $\Im$ be the maximum number of iterations throughout the iteration procedure. The ξth iteration population is expressed as $\Lambda(\xi), \xi = 1, \ldots, \Im$. The implementing object number in each iteration population is kept at Ω for the whole iteration horizon.

Step 2.2: Primary iteration population based on the lowest high wagon–hours consumption.

All Ω implementing objects in the iteration population are put into the reconstruction OBTFM model in turn for solving the objective function value representing the lowest high wagon–hours consumption. The lowest high wagon–hours consumption of Ω implementing objects is obtained. Ω implementing objects are ranked by the ascending order of their lowest high wagon–hours consumption. The size of the primary iteration population is set as θ. Those implementing objects whose precedence is the position in front of θ are selected to form the primary iteration population, which is denoted as $\Lambda_\theta(\xi)$.

Step 2.3: The first updating iteration population based on exchanging coding segments between pairs of implementing objects.

The exchange rate of the coding segment is set as ρ. Two implementing objects are randomly selected from the primary iteration population $\Lambda_\theta(\xi)$ in turn. Then a random number ρ_1 with a uniform distribution between zero and one is generated. If $\rho_1 \le \rho$, two coding segments that are located at the same position of two selected implementing objects are exchanged to generate two new implementing objects. The remaining implementing objects are executed in the mentioned exchanging coding segment operation to obtain the first primary iteration population, which is written as $\Lambda_\theta^1(\xi)$.

Step 2.4: The second updating iteration population based on interchanging coding position order of a single implementing object.

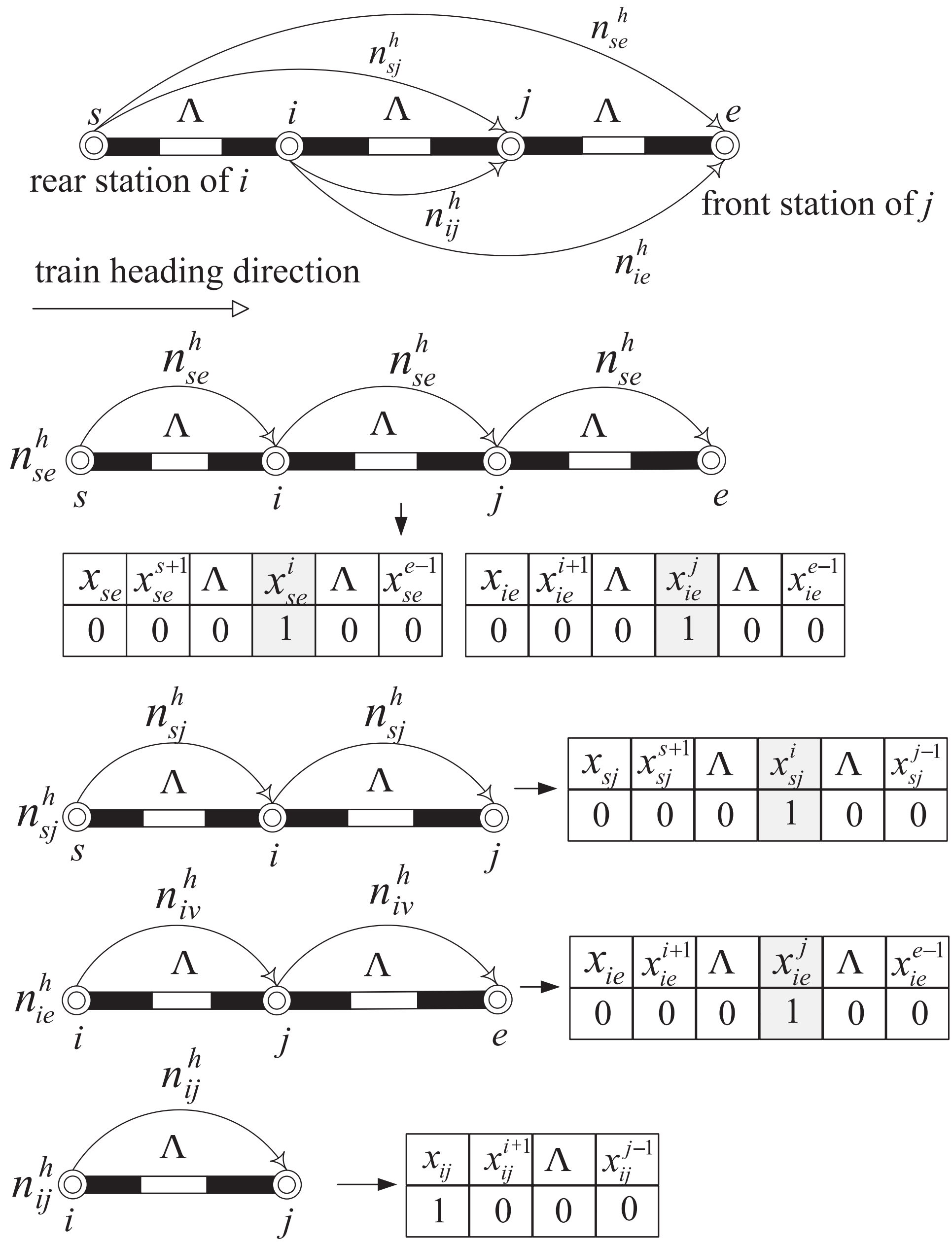

Figure 9. Service wagon flows and coding sequence of block $i \rightarrow j$.

The interchanging rate of coding positions order is set as σ. One implementing object is randomly selected from updating iteration population $\Lambda^1_\theta(\xi)$. Then, a random number σ_1 with a uniform distribution between zero and one is generated. If $\sigma_1 \leq \sigma$, two coding positions located at the same coding segment of the selected implementing object are interchanged to generate one new implementing object. The mentioned interchanging coding position operation executes the remaining implementing objects to obtain the second primary iteration population, denoted as $\Lambda^2_\theta(\xi)$.

Stage 3: Filtering iteration population

Step 3.1: The first filtering iteration population, based on the restriction of the relationship between two decision variables.

All the implementing objects in the second primary iteration population $\Lambda_\theta^2(\xi)$ are put into constraint (6), which means that the relationship between wagon flow reclassification variable x_{ij} and wagon flow reclassification station choice variable x_{ij}^k judges whether the implementing objects are feasible or not. These feasible implementing objects satisfying constraint (6) are collected to establish the first filtering iteration population, denoted as $\Lambda_{\text{filtering}}^1(\xi)$.

Step 3.2: The second filtering iteration population, based on the restriction of classification capacity and classification tracks number.

All implementing objects are selected from the first filtering iteration population $\Lambda_{\text{filtering}}^1(\xi)$ in turn. The selected implementing object and all expanding wagon flow from the expanding data set are put into the classification capacity restriction and classification tracks number restriction of reconstruction OBTFM model to judge whether the classification capacity restriction and classification tracks number restriction are satisfied. All implementing objects satisfying these two constraints are collected to establish the second filtering iteration population, denoted as $\Lambda_{\text{filtering}}^2(\xi)$.

Step 3.3: Updating the ξth iteration optimal implementing object based on wagon–hours consumption.

All implementing objects are selected from the second filtering iteration population $\Lambda_{\text{filtering}}^2(\xi)$ in turn and are put into the equation $\sum_{i\in A}\sum_{j\in A} c_i m x_{ij} + \sum_{i\in A}\sum_{j\in A}\sum_{k\in A} f_{ij}^h x_{ij}^h t_k = G$ to calculate the wagon–hours consumption. The implementing object with the lowest wagon–hours consumption is found and compared with the $(\xi - 1)$th iteration optimal implementing object. The one with the lower wagon–hours consumption is taken as the ξth iteration optimal implementing object, which is denoted as $\Lambda^{\text{best}}(\xi)$.

Step 3.4: Termination criterion.

If the maximum number $\Im$ of iterations is not reached, let $\Lambda(\xi + 1) = \Lambda_{\text{filtering}}^2(\xi)$ and go to Step 2.2. Otherwise, the algorithm is stopped, and the optimal implementing object $\Lambda^{\text{best}}(\Im)$ representing the optimal train formation plan of the OBTFM model is output.

4.3. Heuristic based on covering and joint priority order for HTFM

This section develops an heuristic based on covering and joint priority order for solving the proposed HTFM. First, the procedure for generating a covering two-block train set is developed. Secondly, the procedure for generating a joint two-block train set is given. Finally, the first covering last joint (FCLJ) and first joint last covering (FJLC) strategies are presented. Thus, a hybrid formation plan based on higher total wagon–hours consumption savings is proposed.

Stage 1: Generating a covering two-block train set

Step 1.1: Criterion for generating a covering two-block train.

A covering two-block train formation plan is generated by converting the one-block train formation plan covering one long block and two short blocks. So the criterion for organizing covering two-block trains is that there exist three one-block trains serving one long block and two short blocks.

Step 1.2: Temporary covering two-block train based on a long-block train.

One-block trains are taken out from the optimal implementing object $\Lambda^{\text{best}}(\Im)$ in turn to make a temporary long block, called a temporary long-block train.

Suppose two short-block one-block trains covered by temporary long-block trains can be found from the remaining one-block trains of optimal implementing object $\Lambda^{\text{best}}(\Im)$. In that case, the temporary long-block train is collected together to form a temporary covering two-block train is formed.

If two short-block one-block trains covered by a temporary long-block train cannot be found, another one-block train is taken from the optimal implementing object $\Lambda^{\text{best}}(\Im)$ to see whether it satisfies the condition of covering two short blocks.

Step 1.3: A permanent covering two-block train based on wagon–hours consumption savings.

All temporary covering two-block trains are put into objective function (9) of the HTFM model to calculate the wagon–hours consumption savings by replacing a one-block train with a two-block train. These temporary covering two-block trains with wagon–hours consumption savings are converted to establish permanent two-block trains and are denoted as ϕ_c.

The temporary covering two-block trains without wagon–hours consumption savings are restored to one-block trains.

Step 1.4: Dividing permanent covering two-block train by proportion restrict.

Based on the size of the 1st short-block wagon flow and the 2nd short-block wagon flow, the permanent covering two-block train ϕ_c is divided into two types: permanent covering TBT&LP and permanent covering TBT&UP. In this article, permanent covering TBT&LP and permanent covering TBT&UP are denoted as ϕ_{cl} and ϕ_{cu}, respectively.

Step 1.5: Permanent covering two-block train set.

Three one-block trains converted into permanent two-block trains are removed from optimal implementing object $\Lambda^{\text{best}}(\Im)$. For the remaining one-block trains of $\Lambda^{\text{best}}(\Im)$, the program from Step 1.2 to Step 1.4 is executed until no one-block train satisfies the criterion for generating a covering two-block train. So permanent covering two-block train set can be obtained and denoted as Φ_C.

Stage 2: Generating a joint two-block train set

Step 2.1: Criterion for generating a joint two-block train.

A joint two-block train formation plan is generated by converting the one-block train formation plan jointing two short blocks. So the criterion for organizing a joint two-block train is that there exist two one-block trains serving two short blocks.

Step 2.2: Temporary joint two-block train based on the first short-block.

One-block trains are taken out from the optimal implementing object $\Lambda^{\text{best}}(\Im)$ in turn to make a temporary first short-block, called a temporary first short-block train.

Suppose the other short-block one-block trains adjacent to the temporary first short-block train can be found from the remaining one-block trains of optimal implementing object $\Lambda^{\text{best}}(\Im)$. In that case, it and the temporary first short-block train are collected together to form a temporary joint two-block train is formed.

Suppose the other short-block one-block trains adjacent to the temporary first short-block train cannot be found. In that case, another one-block train is taken from the optimal implementing object $\Lambda^{\text{best}}(\Im)$ to determine whether it satisfies the condition adjacent to the other short block.

Step 2.3: Permanent covering two-block train based on wagon–hours consumption savings.

All temporary joint two-block trains are put into the objective function (9) of the HTFM model to calculate the wagon–hours consumption savings by replacing a one-block train with a two-block train. These temporary joint two-block trains with the wagon–hours consumption savings are converted to establish permanent joint two-block trains and are denoted as ϕ_j.

The temporary joint two-block trains without wagon–hours consumption savings are restored to one-block trains.

Step 2.4: Dividing permanent joint two-block train by the proportion restriction.

Based on the size of the 1st short-block wagon flow and the 2nd short-block wagon flow, the permanent joint two-block train ϕ_j is divided into two types: permanent joint TBT&LP and permanent joint TBT&UP. In this article, permanent joint TBT&LP and permanent joint TBT&UP are denoted as ϕ_{jl} and ϕ_{ju}, respectively.

Step 2.5: Permanent joint two-block train set.

Two one-block trains converted into permanent two-block trains are removed from optimal implementing object $\Lambda^{\text{best}}(\Im)$. For the remaining one-block trains of $\Lambda^{\text{best}}(\Im)$, the program from Step 2.2 to Step 2.4 is executed until no one-block train satisfies the criterion for generating a joint two-block train. So, the permanent joint two-block train set can be obtained and denoted as Φ_J.

Stage 3: Outputting a hybrid formation plan based on a two-block train

Step 3.1: Strategy with first covering last joint (FCLJ).

The program mentioned in Stage 1 is executed to generate a covering two-block train set Φ_C for the optimal implementing object $\Lambda^{best}(\Im)$. Then, the program mentioned in Stage 2 is executed to generate a joint two-block train set Φ_J for the remaining one-blocks. The last remaining one-block is made as an additional one-block set. Finally, a hybrid formation plan with the first covering the last joint is obtained and denoted as $\Phi_{FCLJ} = \{\Phi_C, \Phi_J\}$. The wagon–hours consumption savings of each two-block train in formation plan Φ_{FCLJ} are calculated. Their sum is called the total wagon–hours consumption savings of Φ_{FCLJ}.

Step 3.2: Strategy with first joint last covering (FJLC).

The program mentioned in Stage 2 is executed to generate a joint two-block train set Φ_J for the optimal implementing object $\Lambda^{best}(\Im)$. Then, the program mentioned in Stage 1 is executed to generate a covering two-block train set Φ_C for the remaining one-blocks. The last remaining one-block is made as an additional one-block set. Finally, a hybrid formation plan with the first covering the last joint is obtained and denoted as $\Phi_{FJLC} = \{\Phi_J, \Phi_C\}$. The wagon–hours consumption savings of each two-block train in formation plan Φ_{FJLC} are calculated. Their sum is called the total wagon–hours consumption savings, Φ_{FJLC}.

Step 3.3: Outputting a hybrid formation plan based on higher total wagon–hours consumption savings.

The total wagon–hours consumption savings of Φ_{FCLJ} and Φ_{FJLC} are compared. The hybrid formation with higher total wagon–hours consumption savings is output and denoted as Φ^{best}.

5. Case study

A railway network composed of 12 technical stations is used to evaluate the proposed model and algorithm.

5.1. Experimental data

The railroad network for the experiment includes 12 technical stations, shown in Figure 10. The technical stations are numbered from 1 to 12 to facilitate description. Table 2 shows each technical station's basic information, including accumulation parameters, average classification time per wagon, classification capacity and the number of classification tracks. The number of wagons grouped to form a train is 50. The number of wagons that can be accommodated in each classification track at all technical stations is 200. The relevant parameters of the two-block train are shown in Table 3.

5.2. Expanding statistical wagon flows data

The research reported in this article uses the discrete uniform distribution to describe the wagon flow random variable. The volume of wagon flows in the rail network obeys the uniform distribution listed in Table 4.

Because the wagon flow data obtained throughout the whole planning horizon is insufficient, this article uses the proposed uniform distribution function to expand the 43 wagon flows listed in Table 4. Here, the expanding size of wagon flow data is set as $\tilde{r} = 100$ for each wagon flow.

5.3. Solution obtained by solving the OBTFM model

The research reported in this article first uses the procedure developed in Section 4.1 to transform the probability constraints of the OBTFM model into a deterministic expression. Let the parameter η be 0.95, which means the acceptance rate of the one-block train formation plans with high wagon–hours consumption is 95%. Let the parameters δ and ϵ be 0.90, which means the satisfaction rate of the technical station classification capacity and classification track number is 90%. Then, the IGA method

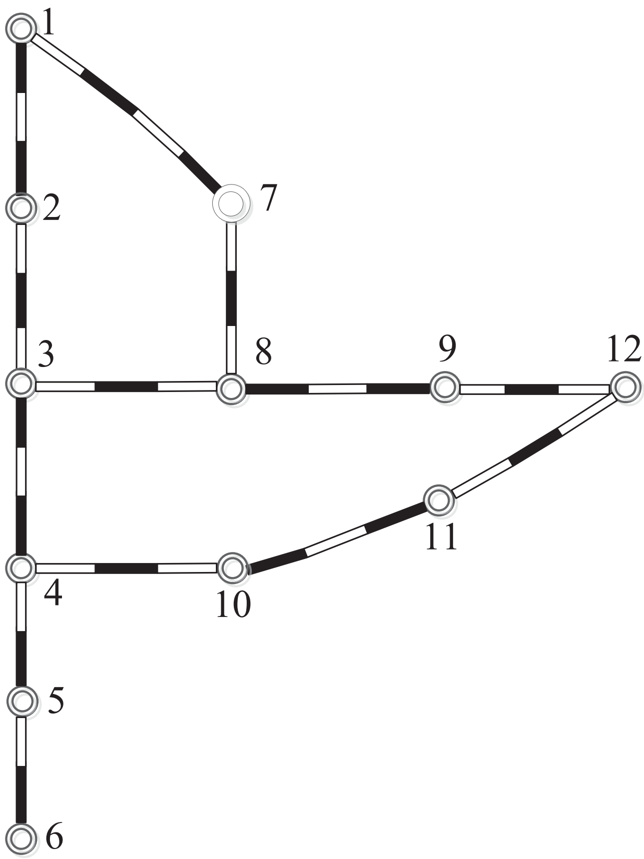

Figure 10. Rail network for the experiment.

Table 2. Parameters of technical stations.

No.	Accumulation parameter	Average classification time per wagon	Classification capacity	Number of tracks
1	11.0	2.0	500	6
2	10.0	2.4	400	5
3	10.5	2.3	450	5
4	10.0	1.9	300	3
5	10.1	2.1	270	4
6	11.6	2.7	250	7
7	10.0	2.2	300	4
8	10.4	2.3	320	8
9	11.1	2.6	380	5
10	10.5	2.1	340	4
11	10.2	2.0	360	4
12	10.3	3.0	300	3

Notes. The number in column 1 denotes the serial number of the technical station. The unit of measure of the classification time is 'hours per wagon'. The unit of measure of the classification capacity is 'wagons per day'.

Table 3. Parameters of the two-block train (units: hours per wagon).

Average additional classification time	Average time delay of detaching wagon group	Average time delay of supplement wagon group	Average time delay of long-block group
0.4	0.3	0.2	0.5

Table 4. Wagon flow data with uniform distribution function.

Wagon flow	Distribution function	Wagon flow	Distribution function	Wagon flow	Distribution function
$n_{1,2}$	$U(40,50)$	$n_{2,8}$	$U(30,36)$	$n_{4,10}$	$U(31,39)$
$n_{1,3}$	$U(42,52)$	$n_{2,9}$	$U(36,44)$	$n_{4,11}$	$U(36,44)$
$n_{1,4}$	$U(45,55)$	$n_{2,10}$	$U(41,51)$	$n_{4,12}$	$U(41,51)$
$n_{1,5}$	$U(43,53)$	$n_{2,11}$	$U(39,47)$	$n_{5,6}$	$U(40,50)$
$n_{1,6}$	$U(56,68)$	$n_{2,12}$	$U(36,44)$	$n_{7,8}$	$U(42,52)$
$n_{1,7}$	$U(47,57)$	$n_{3,4}$	$U(52,64)$	$n_{7,9}$	$U(30,36)$
$n_{1,8}$	$U(43,53)$	$n_{3,5}$	$U(44,54)$	$n_{7,12}$	$U(47,57)$
$n_{1,9}$	$U(59,73)$	$n_{3,6}$	$U(45,55)$	$n_{8,9}$	$U(33,41)$
$n_{1,10}$	$U(31,39)$	$n_{3,8}$	$U(47,57)$	$n_{8,12}$	$U(38,46)$
$n_{1,11}$	$U(36,44)$	$n_{3,9}$	$U(49,61)$	$n_{9,12}$	$U(32,40)$
$n_{1,12}$	$U(48,58)$	$n_{3,10}$	$U(48,58)$	$n_{10,11}$	$U(36,44)$
$n_{2,3}$	$U(34,42)$	$n_{3,11}$	$U(42,52)$	$n_{10,12}$	$U(39,47)$
$n_{2,4}$	$U(36,44)$	$n_{3,12}$	$U(40,50)$	$n_{11,12}$	$U(34,42)$
$n_{2,5}$	$U(31,39)$	$n_{4,5}$	$U(42,52)$	–	–
$n_{2,6}$	$U(40,48)$	$n_{4,6}$	$U(45,55)$	–	–

Notes. The symbol U in columns 2, 4 and 6 denotes the uniform distribution function and the numbers express the upper and lower bounds of the uniform distribution.

Table 5. Solution obtain by solving the OBTFM model with the IGA.

No.	Block section	Consolidated wagon flow	Wagon volume	One-block train frequency
1	1–2	$n_{1,2}+n_{1,5}+n_{1,11}$	138	2.76
2	1–3	$n_{1,3}+n_{1,6}$	104	2.08
3	1–4	$n_{1,4}+n_{1,10}$	93	1.86
4	1–7	$n_{1,7}+n_{1,8}$	98	1.96
5	1–9	$n_{1,9}+n_{1,12}$	118	2.36
6	2–3	$n_{2,3}+n_{2,6}$	81	1.62
7	2–4	$n_{2,4}+n_{1,5}+n_{2,5}+n_{1,11}+n_{2,11}$	205	4.10
8	2–8	$n_{2,8}+n_{2,9}$	71	1.42
9	2–10	$n_{2,10}$	51	1.02
10	2–12	$n_{2,12}$	43	0.86
11	3–4	$n_{3,4}$	63	1.26
12	3–5	$n_{3,5}+n_{1,6}+n_{2,6}+n_{3,6}$	205	4.10
13	3–8	$n_{3,8}$	50	1.00
14	3–9	$n_{3,9}+n_{3,12}$	100	2.00
15	3–10	$n_{3,10}+n_{3,11}$	98	1.96
16	4–5	$n_{4,5}+n_{1,5}+n_{2,5}$	129	2.58
17	4–6	$n_{4,6}$	52	1.04
18	4–10	$n_{4,10}+n_{1,10}$	74	1.48
19	4–11	$n_{4,11}+n_{1,11}+n_{2,11}+n_{4,12}$	169	3.38
20	5–6	$n_{5,6}+n_{1,6}+n_{2,6}+n_{3,6}$	202	4.04
21	7–8	$n_{7,8}+n_{1,8}+n_{7,9}+n_{7,12}$	182	3.64
22	8–9	$n_{8,9}+n_{2,9}+n_{7,9}+n_{7,12}+n_{8,12}$	198	3.96
23	9–12	$n_{9,12}+n_{1,12}+n_{3,12}+n_{7,12}+n_{8,12}$	208	4.16
24	10–11	$n_{10,11}+n_{3,11}$	83	1.66
25	10–12	$n_{10,12}$	43	0.86
26	11–12	$n_{11,12}+n_{4,12}$	89	1.78

Notes. The number in column 1 denotes the serial number of the technical station. Wagon volume in column 4 denotes the number of wagons carried by the one-block train per day in the block section. Train frequency in column 5 denotes the number of one-block trains operating per day in the block section.

proposed in Section 4.1 is programmed by MATLAB®. The parameters of the IGA approach are set as follows: iteration population size $\Omega = 100$; maximum number of iterations $\theta = 100$; exchanging rate of coding segments $\rho = 0.9$; and interchanging rate of coding position order $\sigma = 0.1$.

The OBTFM model is solved by executing the IGA program. The final one-block train formation plan is illustrated in Table 5 and Figure 11. Table 4 lists the block section, consolidated wagon flow, wagon volume and train frequency. Figure 11 shows a one-block train formation plan. The total wagon–hours consumption is 15,682.

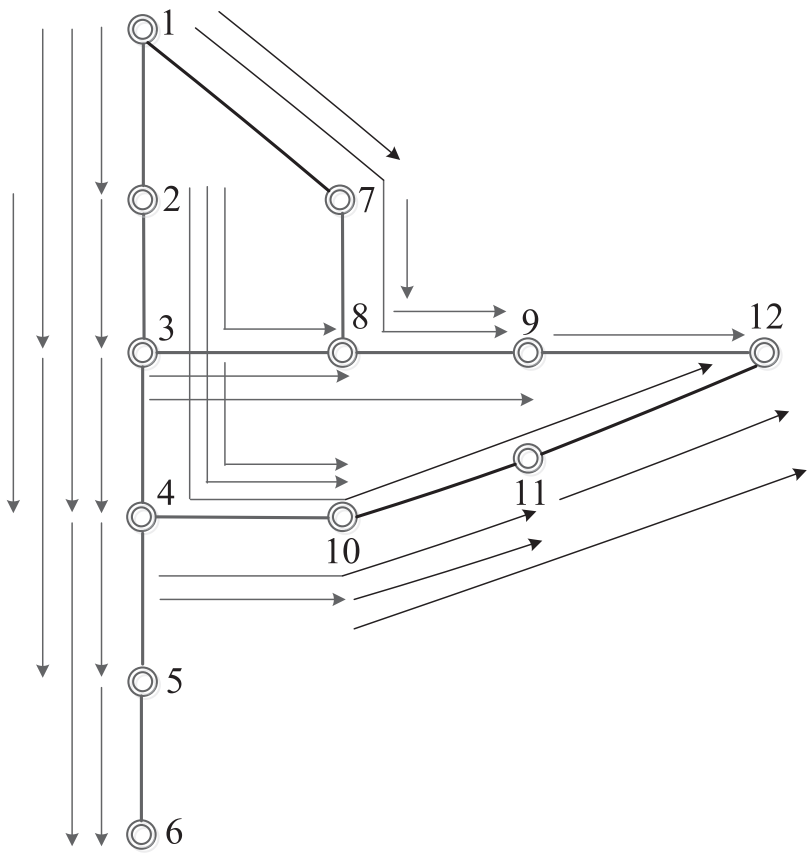

Figure 11. One-block train formation plan obtained by solving the OBTFM model with the IGA.

To verify the efficiency and validity of the IGA, this article also uses the optimization software LINGO™17.0 to solve the problem on the same computer. The final one-block train formation plan obtained by LINGO17.0 is illustrated in Table 6 and Figure 12. Table 5 lists the block section, consolidated wagon flow, wagon volume and train frequency. Figure 12 shows a one-block train formation plan. The total wagon–hours consumption is 21,540.4. The research reported in this article shows that the wagon–hours consumption of the solution obtained by LINGO is significantly greater than that of the solution by the IGA. Therefore, using the improved genetic algorithm to solve the OBTFM model can obtain a better solution.

5.4. Solution obtained by solving the HTFM model

The research reported in this article uses an heuristic based on covering and joint priority order as provided in Section 4.2 to solve the HTFM model. The two-block train of the hybrid train formation plan obtained by solving the HTFM model is also displayed in Table 7, which lists the two-block section, wagon–hours consumption savings and two-block train type.

The formation plan based on covering TBT&UP is illustrated in Figures 13 to 17. In Figure 13, covering TBT&UP combines three one-block trains serving one long block and two short blocks, *i.e.* train $\{n_{3,9} + n_{3,12}\}$ serving long block $\{3 \rightarrow 9\}$, train $\{n_{3,8}\}$ serving the 1st short block $\{3 \rightarrow 8\}$ and train $\{n_{8,9} + n_{2,9} + n_{7,9} + n_{7,12} + n_{8,12}\}$ serving the 2nd short block $\{8 \rightarrow 9\}$. Technical station 8 is a block-exchange station. When the two-block train arrives at block-exchange station 8, the 1st short-block wagon flow will be replaced by the 2nd short-block wagon flow of the same size

Table 6. Solution obtained by solving the OBTFM model with LINGO.

No.	Block section	Consolidated wagon flow	Wagon volume	One-block train frequency
1	1–2	$n_{1,2} + n_{1,3} + n_{1,4}$	144	2.88
2	1–5	$n_{1,5}$	50	1.00
3	1–6	$n_{1,6}$	62	1.24
4	1–7	$n_{1,7}$	53	1.06
5	1–8	$n_{1,8}$	45	0.90
6	1–9	$n_{1,9}$	65	1.03
7	1–10	$n_{1,10}$	39	0.78
8	1–11	$n_{1,11}$	40	0.80
9	1–12	$n_{1,12}$	53	1.06
10	2–3	$n_{2,3} + n_{1,3}$	78	1.56
11	2–4	$n_{2,4} + n_{1,4} + n_{2,5}$	130	2.60
12	2–6	$n_{2,6}$	45	0.90
13	2–8	$n_{2,8}$	32	0.64
14	2–9	$n_{2,9}$	39	0.78
15	2–10	$n_{2,10}$	51	1.02
16	2–11	$n_{2,11}$	39	0.78
17	2–12	$n_{2,12}$	43	0.86
18	3–4	$n_{3,4}$	63	1.26
19	3–5	$n_{3,5}$	52	1.04
20	3–6	$n_{3,6}$	46	0.92
21	3–8	$n_{3,8}$	50	1.00
22	3–9	$n_{3,9}$	53	1.06
23	3–10	$n_{3,10}$	55	1.10
24	3–11	$n_{3,11}$	43	0.86
25	3–12	$n_{3,12}$	47	0.94
26	4–5	$n_{4,5} + n_{2,5}$	79	1.58
27	4–6	$n_{4,6}$	52	1.04
28	4–10	$n_{4,10}$	35	0.70
29	4–11	$n_{4,11}$	42	0.84
30	4–12	$n_{4,12}$	48	0.96
31	5–6	$n_{5,6}$	49	0.98
32	7–8	$n_{7,8}$	51	1.02
33	7–9	$n_{7,9}$	32	0.64
34	7–12	$n_{7,12}$	54	1.08
35	8–9	$n_{8,9}$	35	0.70
36	8–12	$n_{8,12}$	38	0.76
37	9–12	$n_{9,12}$	38	0.76
38	10–11	$n_{10,11}$	40	0.80
39	10–12	$n_{10,12}$	43	0.86
40	11–12	$n_{11,12}$	41	0.82

Note. The number in column 1 denotes the serial number of the technical station.

. Because the size of the 1st short-block wagon flow ($n_{3,8}$) that originated at technical station 3 is less than the size of the 2nd short-block wagon flow ($n_{8,9} + n_{2,9} + n_{7,9} + n_{7,12} + n_{8,12}$) that originated at block-exchange station 8, the type of two-block train is a covering two-block train with an unlimited proportion. The remaining wagon flow ($n_{8,9} + n_{2,9} + n_{7,9} + n_{7,12} + n_{8,12} - n_{3,8}$) will be collected together to form an additional one-block train serving the 2nd short block at block-exchange station 8.

In Figure 14, covering TBT&UP combines three one-block trains serving one long block and two short blocks, *i.e.* train $\{n_{10,12}\}$ serving long block $\{10 \to 12\}$, train $\{n_{10,11} + n_{3,11}\}$ serving the 1st short block $\{10 \to 11\}$ and train $\{n_{11,12} + n_{4,12}\}$ serving the 2nd short block $\{11 \to 12\}$. Technical station 11 is a block-exchange station. When the two-block train arrives at block-exchange station 11, the 1st short-block wagon flow will be replaced by the 2nd short-block wagon flow ofthe same size . Because the size of the 1st short-block wagon flow ($n_{10,11} + n_{3,11}$) that originated at technical station 10 is less than the size of the 2nd short-block wagon flow ($n_{11,12} + n_{4,12}$) that originated at block-exchange station 11, the type of two-block train is a covering two-block train with

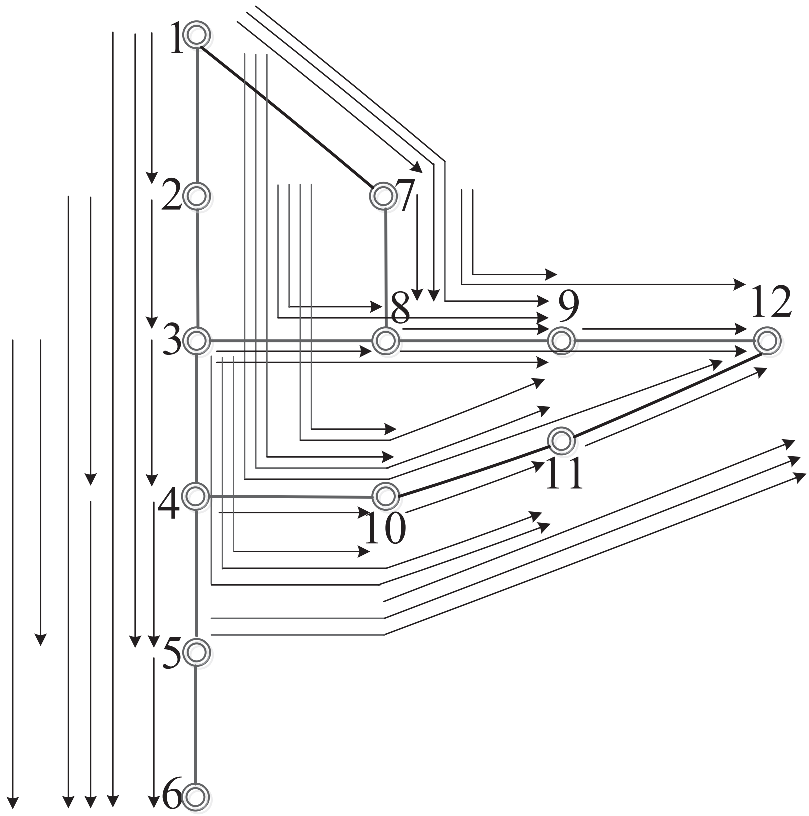

Figure 12. One-block train formation plan obtained by solving the OBTFM model with software LINGO™ 17.0.

Table 7. Two-block train obtained by solving the HTFM model.

No.	Two-block	Wagon–hours consumption savings	Two-block train type
1	3–8–9	366.5	Covering TBT&UP
2	10–11–12	354.1	Covering TBT&UP
3	1–2–4	314.9	Covering TBT&UP
4	3–4–10	312.9	Covering TBT&UP
5	4–5–6	273.3	Covering TBT&UP
6	1–9–12	23.1	Joint TBT&UP

Notes. The 'Two-block' column 2 figures denote the departure station, block-exchange station and destination station.

an unlimited proportion. The remaining wagon flow $(n_{11,12} + n_{4,12} - n_{10,11} - n_{3,11})$ will be collected together to form an additional one-block train serving the 2nd short block at block-exchange station 11.

In Figure 15, covering TBT&UP combines three one-block trains serving one long block and two short blocks, *i.e.* train $\{n_{1,4} + n_{1,10}\}$ serving long block $\{1 \rightarrow 4\}$, train $\{n_{1,2} + n_{1,5} + n_{1,11}\}$ serving the 1st short block $\{1 \rightarrow 2\}$ and train $\{n_{2,4} + n_{1,5} + n_{2,5} + n_{1,11} + n_{2,11}\}$ serving the 2nd short block $\{2 \rightarrow 4\}$. Technical station 2 is a block-exchange station. When the two-block train arrives at block-exchange station 2, the 1st short-block wagon flow will be replaced by the 2nd short-block wagon flow of the same size . Because the size of the 1st short-block wagon flow $(n_{1,2} + n_{1,5} + n_{1,11})$ that originated at technical station 1 is less than the size of the 2nd short-block

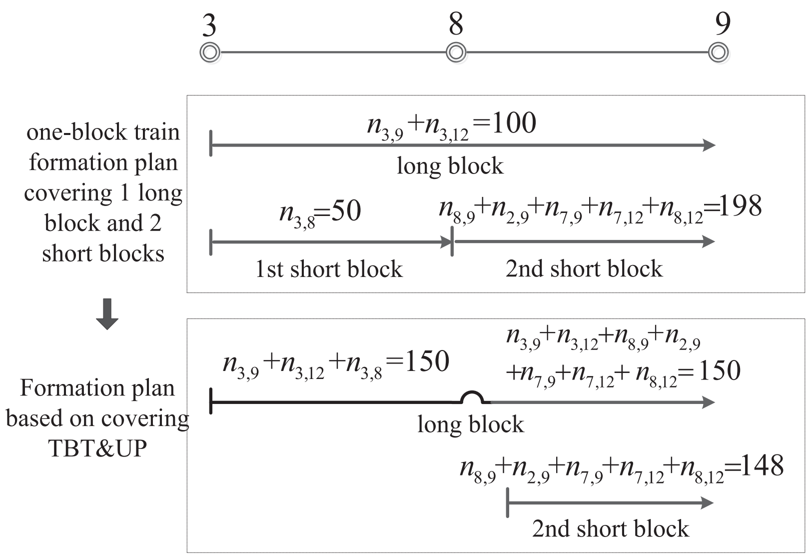

Figure 13. Formation plan based on covering TBT&UP combining long block 3-9, short block 3-8 and 8-9.

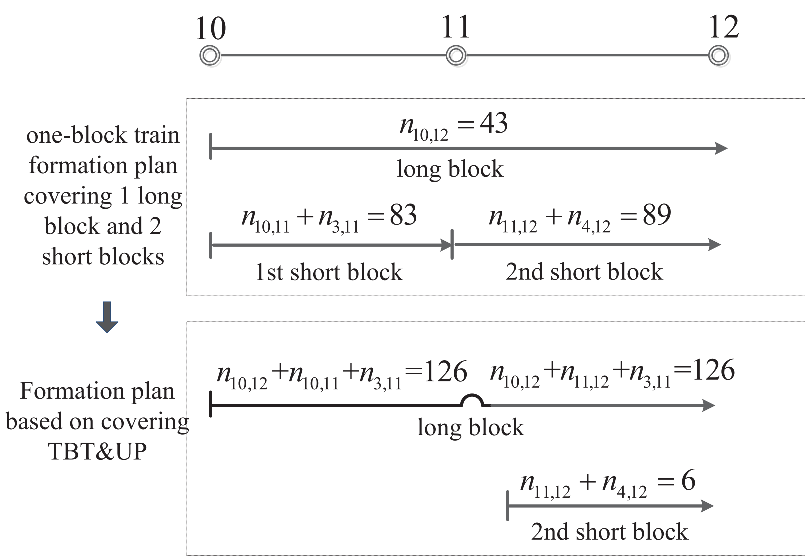

Figure 14. Formation plan based on covering TBT&UP combining long block 10-12, short block 10-11 and 11-12.

wagon flow ($n_{2,4}+n_{1,5}+n_{2,5}+n_{1,11}+n_{2,11}$) that originated at block-exchange station 2, the type of two-block train is a covering two-block train with an unlimited proportion. The remaining wagon flow ($n_{2,4}+n_{1,5}+n_{2,5}+n_{1,11}+n_{2,11}-n_{1,2}-n_{1,5}-n_{1,11}$) will be collected together to form an additional one-block train serving the 2nd short block at block-exchange station 2.

In Figure 16, covering TBT&UP combines three one-block trains serving one long block and two short blocks, *i.e.* train $\{n_{4,6}\}$ serving long block $\{4 \rightarrow 6\}$, train $\{n_{4,5}+n_{1,5}+n_{2,5}\}$ serving the 1st short block $\{4 \rightarrow 5\}$ and train $\{n_{5,6}+n_{1,6}+n_{2,6}+n_{3,6}\}$ serving the 2nd short block $\{5 \rightarrow 6\}$. Technical station 5 is a block-exchange station. When the two-block train arrives at block-exchange station 5, the 1st short-block wagon flow will be replaced by the 2nd short-block wagon flow of the same size .

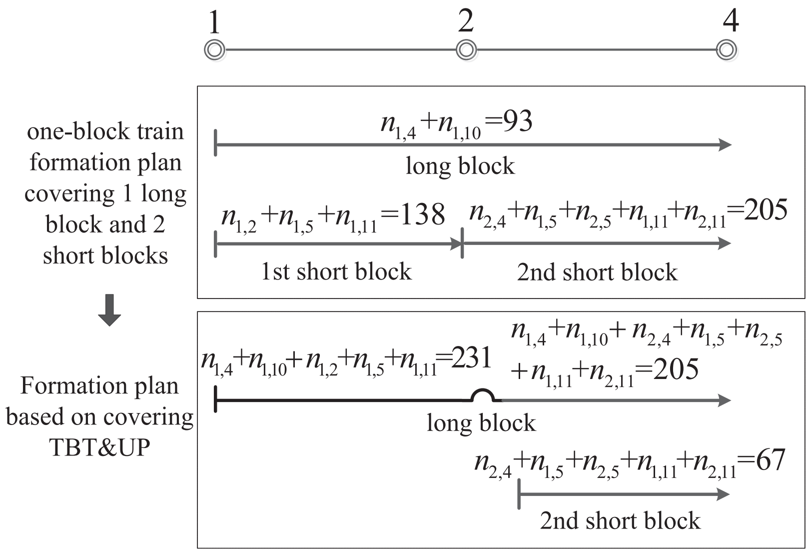

Figure 15. Formation plan based on covering TBT&UP combining long block 1-4, short block 1-2 and 2-4.

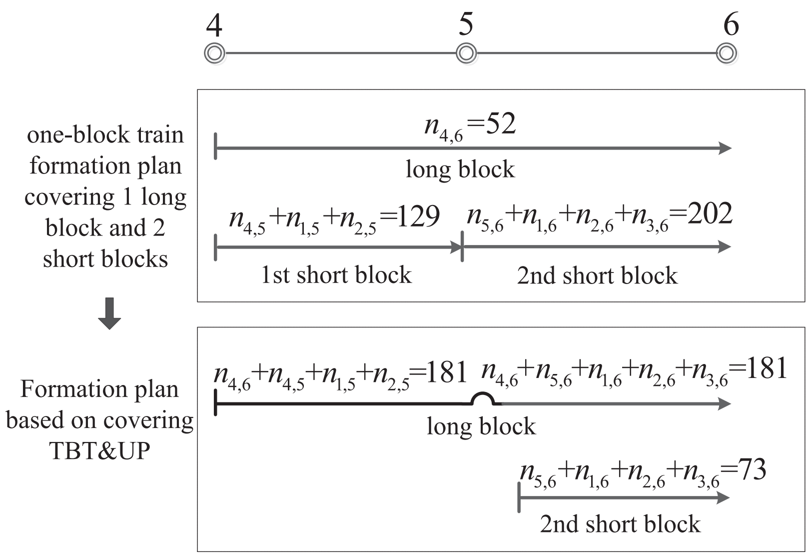

Figure 16. Formation plan based on covering TBT&UP combining long block 4-6, short block 4-5 and 5-6.

Because the size of the 1st short-block wagon flow ($n_{4,5} + n_{1,5} + n_{2,5}$) that originated at technical station 4 is less than the size of the 2nd short-block wagon flow ($n_{5,6} + n_{1,6} + n_{2,6} + n_{3,6}$) that originated at block-exchange station 2, the type of two-block train is a covering two-block train with an unlimited proportion. The remaining wagon flow ($n_{5,6} + n_{1,6} + n_{2,6} + n_{3,6} - n_{4,5} - n_{1,5} - n_{2,5}$) will be collected together to form an additional one-block train serving the 2nd short block at block-exchange station 5.

In Figure 17, covering TBT&UP combines three one-block trains serving one long block and two short blocks, *i.e.* train $\{n_{3,10} + n_{3,11}\}$ serving long block $\{3 \rightarrow 10\}$, train $\{n_{3,4}\}$ serving the 1st short block $\{3 \rightarrow 4\}$ and train $\{n_{4,10} + n_{1,10}\}$ serving the 2nd short block $\{4 \rightarrow 10\}$. Technical station 4 is a

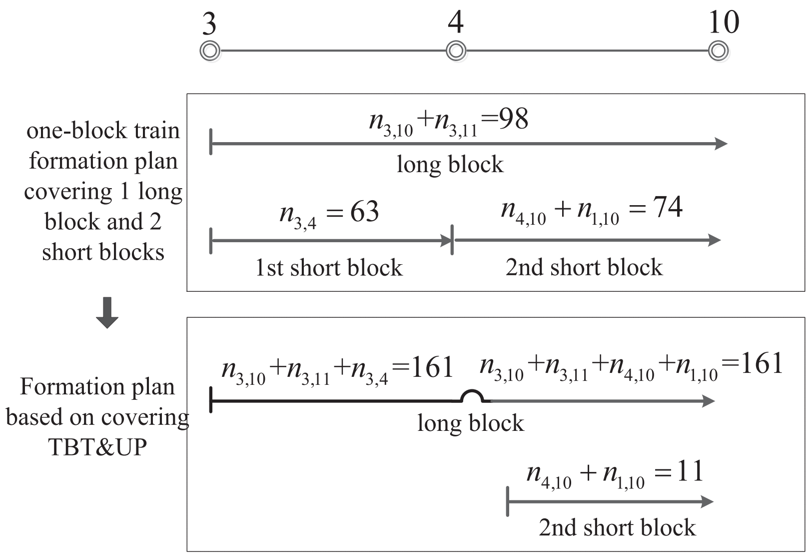

Figure 17. Formation plan based on covering TBT&UP combining long block 3-10, short block 3-4 and 4-10.

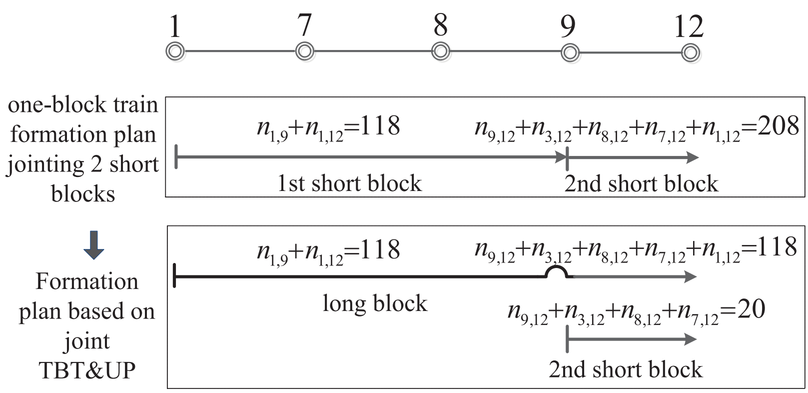

Figure 18. Formation plan based on joint TBT&UP obtained by solving the HTFM model.

block-exchange station. When the two-block train arrives at block-exchange station 4, the 1st short-block wagon flow will be replaced by the 2nd short-block wagon flow of the same size . Because the size of the 1st short-block wagon flow ($n_{3,4}$) that originated at technical station 3 is less than the size of the 2nd short-block wagon flow ($n_{4,10} + n_{1,10}$) that originated at block-exchange station 4, the type of two-block train is a covering two-block train with an unlimited proportion. The remaining wagon flow ($n_{4,10} + n_{1,10} - n_{3,4}$) will be collected together to form an additional one-block train serving the 2nd short block at block exchange station 4.

The formation plan based on joint TBT&UP is shown in Figure 18. Joint TBT&UP combines two one-block trains serving two short blocks, *i.e.* train $\{n_{1,9} + n_{1,12}\}$ serving the 1st short block $\{1 \to 9\}$ and train $\{n_{9,12} + n_{3,12} + n_{8,12} + n_{7,12} + n_{1,12}\}$. Technical station 9 is a block-exchange station. When the two-block train arrives at block-exchange station 9, the 1st short-block wagon flow will be replaced by the 2nd short-block wagon flow of the same size . Because the size of the 1st short-block wagon flow ($n_{1,9} + n_{1,12}$) that originated at technical station 2 is less than the size of the 2nd short-block wagon flow ($n_{9,12} + n_{3,12} + n_{8,12} + n_{7,12} + n_{1,12}$) that originated

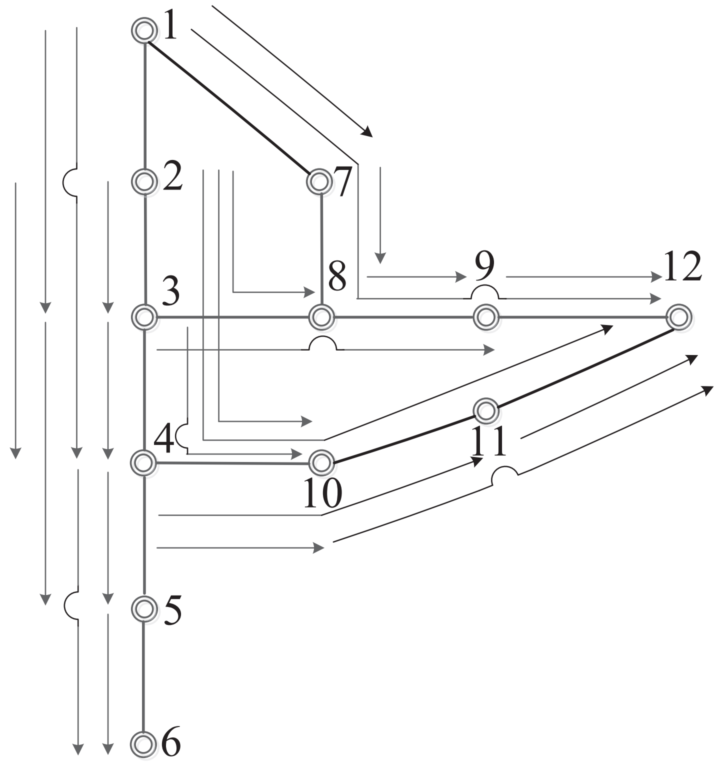

Figure 19. A hybrid train formation plan obtained by solving the HTFM model.

at block-exchange station 9, the type of two-block train is a joint two-block train with an unlimited proportion. The remaining wagon flow ($n_{9,12} + n_{3,12} + n_{8,12} + n_{7,12} + n_{1,12} - n_{1,9} - n_{1,12}$) will be collected to form an additional one-block train serving the 2nd short block at block-exchange station 9.

The hybrid train formation plan is illustrated in Figure 19. The total wagon–hours consumption of the hybrid train formation plan is 14,037.2. It is obviously less than the wagon–hours consumption of a one-block formation plan. Therefore, a hybrid train formation plan can provide more efficient organizational work than a one-block one.

6. Conclusions

In this article, the study aims to develop the formulation and solution of a train formation plan under wagon demand fluctuation (TFP&WDF). First, a one-block train formation model (OBTFM) for the TFP&WDF problem is developed. The objective function minimizes one-block train accumulation time at the departure technical station and reclassification time at the middle technical station. Because the OBTFM model includes three probability constraints, it is difficult to solve. For this reason, this article provides a procedure for transforming probability constraints into deterministic expressions. Then, an improved genetic algorithm (IGA) for solving the OBTFM model is presented.

In order to acquire a more highly efficient train formation plan, this article tries to convert some one-block trains to two-block trains. The research reported in this article proposes four types of two-block train: Covering TBT&LP, Covering TBT&UP, Joint TBT&LP and Joint TBT&UP. The wagon–hours savings generated by replacing one-block trains with the four different types of two-block train is systematically analysed. In terms of saved wagon–hours consumption, a hybrid train formation model integrating one-block and two-block trains for TFP&WDF is developed. An heuristic based on covering and joint priority order for solving the proposed HTFM is given.

In the case study section, this article tests the model and solving approach by a railway network composed of nine technical stations in China. The wagon–hours consumption of the one-block train formation plan is 15,682. The wagon–hours consumption of the hybrid train formation plan is 14,037.2. The latter reduces wagon–hours consumption by 10.5%.

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Disclosure statement

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Data availability statement

The data that support the findings of this study are available on request from the corresponding author.

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Appendix. Notation

Sets

A Set of technical stations in a rail network. Define $A = \{a \mid a = 1, 2, \ldots, \tilde{a}\}$, where $\tilde{a}$ is the total number of technical stations.

P_a Set of the backward technical stations of the station in a rail network. Define $P_a = \{p \mid p = 1, 2, \ldots, \tilde{p}\}$, where $\tilde{p}$ is the total number of backward technical stations of station a.

Q_a Set of the forward technical stations of the station in a rail network. Define $Q_a = \{q \mid q = 1, 2, \ldots, \tilde{q}\}$, where $\tilde{q}$ is the total number of forward technical stations of station a.

H Set of statistical time periods through the planning horizon. Define $H = \{h \mid h = 1, \ldots, \tilde{h}\}$, where $\tilde{h}$ is the number of statistical time periods.

Parameters

b_a Classification capacity of technical station a (wagons per day).

λ_a Utilization level of classification capacity of technical station a.

d_a The number of classification tracks at technical station a.

μ_a The number of wagons that can be accommodated in each classification track at technical station a.

c_a Accumulation parameter of technical station a, denoting the time consumption to form a train.

t_a Average classification time per wagon arising from one-block train at technical station a (hours per wagon).

t_a^b Average classification time per wagon arising from two-block train at technical station a (hours per wagon).

t_a^c Average time delay of a detaching wagon group, which means the average time consumption per wagon of a detaching wagon group. The time consumption is generated by a detaching wagon group when it is dropped from a two-block train at block-exchange station a (hours per wagon).

t_a^z Average time delay of a supplement wagon group, which means the average time consumption per wagon of the supplement wagon group. The time consumption is generated by the supplement wagon group when it is attached to a two-block train at block-exchange station a (hours per wagon).

t_a^l Average time delay of a long block group, which means the average time consumption per wagon of the long block group. Time consumption is the total time delay, which is the sum of dropping the detaching wagon group and attaching the supplement wagon group at block exchange station a (hours per wagon).

m The number of wagons grouped to form a train.

n_{ij}^h Original O-D wagon flow volume from i to j, *i.e.* the number of wagon flows that originate at technical station i and are destined for technical station j in the statistical time period h.

f_{ij}^h Actual wagon flow volume from i to j, *i.e.* the number of wagon flows that originate or reclassify at technical station i and are destined for technical station j in the statistical time period h. It is composed of two parts. The first part is the original wagon flow that originates at technical station i and is destined for technical station j. The second part is the reclassification of wagon flow that originates at the rear technical station of technical station i, reclassifies at technical station i, and is destined for technical station j.

g_{ij}^h Service wagon flow volume from i to j, *i.e.* the number of wagon flows that originate or are reclassified at technical station i, are reclassified or are destined for technical station j in the statistical time period h. It is composed of three parts. The first part is the original wagon flow that originates at technical station i and is destined for technical station j. The second part is the reclassification of a wagon flow that originates at the rear technical station of technical station i, is reclassified at technical station i, and is destined for technical station j. The third part is the reclassification of a wagon flow that originates at technical station i, is reclassified at technical station j, and is destined for the front technical station of technical station j.

Decision variables

x_{ij} Wagon flow reclassification variable. Its value is one if the wagon flows from stations i to j and is not reclassified on its itinerary. Otherwise, it is zero.

x_{ij}^k Wagon flow reclassification station choice variable. Its value is one if the technical station k is the station where the wagon flow from stations i to j is first reclassified. Otherwise, it is zero.

y_{ij}^k One-block or two-block train choice variable. Its value is one if the wagon flow from i to j is organized into a two-block train at block-exchange station k. Its value is zero if the wagon flow from i to j is organized into a one-block train.

u_{ij}^k Covering two-block train or joint two-block train judging variable. Its value is one if a two-block train with origin station i, block-exchange station k and destination station j covers a two-block train. If it is a joint two-block train, its value is zero.

v_{ij}^k Two-block train with a limited proportion or an unlimited proportion judging variable. Its value is one if a two-block train with origin station i, block-exchange station k and destination station j is a two-block train with an unlimited proportion. If it is a two-block train with a limited proportion, its value is zero.