



An adaptive large neighborhood search based approach for the vehicle routing problem with zone-based pricing

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ARTICLE INFO

Keywords:

Vehicle routing problem
Zone-based pricing
Adaptive large neighborhood search
Local search

ABSTRACT

In the classical vehicle routing problems, the objective function is usually to minimize the transportation cost without considering the pricing decision-making. However, in some practical applications, such as riding-share services and fast-food delivery services, logistics companies aim to maximize profits, involving decision-making issues closely related to the pricing and route planning. This paper designs an adaptive large neighborhood search (ALNS) based approach to solve a vehicle routing problem with zone-based pricing (VRPZ). In the studied problem, the customers in the same zone share the same price for transportation, and each customer could accept or reject the price given by the logistics company. On the one hand, a too-high pricing strategy may reduce the number of served customers; on the other hand, a too-low pricing strategy may result in low earnings for logistics companies. VRPZ optimizes the routing planning and seeks an appropriate pricing strategy to maximize the total profit, which is the difference value of total earnings and costs. Considering that the studied problem is a computational challenge, we have developed an algorithm incorporating the local search into ALNS, particularly nested with two varying pricing operators: price++ and price--, to solve the problem. The proposed algorithm has been compared with pure ALNS, variable neighborhood search (VNS) and Simulated Annealing (SA) by solving 472 benchmark instances in the published work. The compared results, which are validated by the statistic tests, highlight the efficiency and effectiveness of the proposed algorithm. To our best knowledge, this is the first heuristic to achieve a good performance in solving the benchmark instances. This study is significant for the retail industry to find a reasonable logistics pricing strategy and improve operational efficiency, as well as lays a foundation for developing a decision support system.

1. Introduction

Vehicle Routing Problem (VRP) is a practical and critical issue in a wide range of applications, such as logistics system design for the retail industry (Yang et al., 2020), home healthcare routing planning (Shi et al., 2019), supply chain optimization (Al Theeb et al., 2020), city logistics (Liu et al., 2020), etc. The basic VRP can be defined as seeking the lowest-cost distribution routes from a depot to a geographically dispersed set of customers, constrained by the capacity of each vehicle. Hence, the basic VRP is also called Capacitated VRP (CVRP). VRP has many variations due to the various operational rules and constraints in real applications (Ghannadpour and Zandiyeh, 2020). In the fundamental VRP problem, we assume that the transport price is directly proportional to the distribution distance. However, in the actual logistics operation system, it is easy to find that the distribution price often presents a stepped rather than linear relationship. This

pricing model can often be attributed to the zonal pricing of logistics. This study focuses on the VRP with the zone pricing (VRPZ) strategy.

In VRPZ, similar to CVRP, vehicles start from a depot and then serve a group of geographically dispersed customers, which are divided into different zones. But, the difference (see Table 1 for more details) between the VRPZ and CVRP is that the price for the transportation shares the heterogeneous strategies; namely, the customer located in the different zones are paying the different prices for the delivery man. Customers can choose to receive or reject the logistics company's offered price by comparing the psychologically expected price and quotation provided by the company. If the price raised by the logistics company is higher than the customer's threshold price, the customer rejects this service. In our work, the lowest psychologically expected price is defined as a threshold value of a customer.

VRPZ has many practical applications. For example, the fast-food delivery industry is a growing industry that takes orders from restaurants and then delivers them to customers' houses or working offices.

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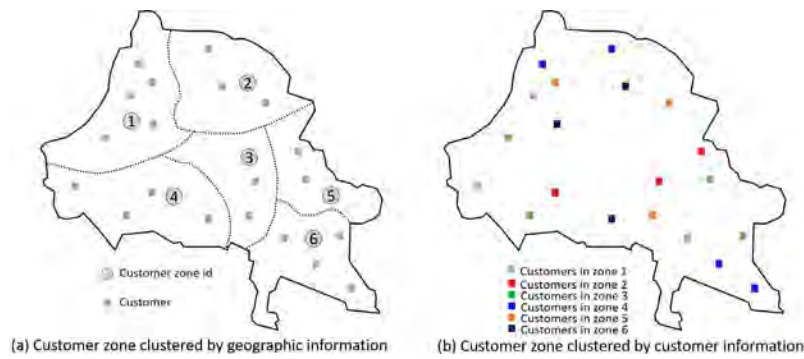


Fig. 1. A toy problem with zone-based delivery.

Table 1
The comparison between CVRP and VRPZP.

	VRPZP	CVRP
model input	customers divided by regions, demand, cost matrix, expected price	customers, demand, cost matrix
objective	revenue=earnings-costs	costs
number of regions	greater than 1	1
serviced customers	all the customers must be serviced	only the selected customers are serviced.
main operators	inter-route, intra-route, pricing changing (complicated)	inter-route, intra-route

Fast food delivery companies must arrange daily delivery schedules to serve customers who order fast food online. Different food delivery companies have different pricing strategies. And each customer prefers to select the lower delivery price; one may choose to reject the delivery service when the price is higher than the threshold of the expected price. As depicted in Fig. 1, customers are divided into different regions and waiting for the delivery service.

Afsar et al. (2021) made the earliest attempt to formulate a VRPZ by proposing a mixed-integer programming model. The authors have developed a branch & price algorithm to solve this model. To demonstrate the efficiency of the proposed algorithm, they created a few groups of benchmark instances and reported the lower bound and the gap for each instance. However, as VRP is an NP-hard problem (Lenstra and Kan, 1981), there is no doubt that VRP with zone pricing also has the problem of computational challenge. As shown in the results of Afsar et al. (2021), the computational time is very large. This motivates us to attempt to develop a heuristic algorithm to solve the model.

Obviously, VRPZ can be viewed as an extension of VRP. Since VRP is a well-known NP-hard combinatorial optimization problem (Lenstra and Kan, 1981), and only relatively small-scale instances of VRP can achieve an optimal solution in a short computational time (Desaulniers et al., 2008). In this study, we focus on designing a heuristic approach, namely ALNS, to solve VRPZ.

The pricing strategy in the model makes this problem difficult to be solved by traditional neighborhood search heuristic algorithms. For example, if the price in a zone changes, the served customers' set may also change due to the fact that some customers may reject or re-accept the price. Specifically, when the price increases, customers in the same zone may reject the service and no longer stay on the service list; while the price decreases, some customers may accept the price and add to the service list again. Therefore, the new decision variables, the price at each zone, significantly impact the service and routing lists. Heuristics based on constraint programming have been designed to solve this problem because constraint programming provides a simple way to express the pricing constraints. To achieve a high-quality solution in a short time, we have designed two particular varying pricing operators to address the above difficulties in this study. Then, based on these operators, we construct an efficient hybrid ALNS algorithm to solve this VRPZ problem. Numerical experiments on a set of benchmark instances show that the proposed algorithm can obtain satisfactory solutions in an acceptable execution time.

We observe that, unlike the basic CVRP, which have been studied intensively, few meta-heuristic algorithms for solving VRPZP have been investigated yet. The branch and price algorithm has been proposed by Afsar et al. (2021), providing us with a lower bound, but the computational time still needs to be improved when applied to the practice. Considering that the routing planning problem is an immediate decision, it is crucial to develop a fast and effect algorithm. Inspired by the above consideration, we present the first meta-heuristic algorithm for solving VRPZP, and the algorithm optimizes the routing planning and the pricing simultaneously. In the proposed algorithm, local search is embedded into the framework of ALNS with a new procedure, and the operators of local search are applied with the adaptive mechanism of ALNS. In particular, consider the features of the studied problem, the proposed algorithm features a pricing-changing strategy and design the new removal operator of ALNS. We assess the algorithm with the benchmark instances designed by Afsar et al. (2021), and finally, the experimental results demonstrate the effectiveness and efficiency of the proposed algorithm.

The main contribution of this study is to design a hybrid metaheuristic algorithm, namely ALNS, to solve the problem by combining the characteristics of the model. The results are of great significance for obtaining high-quality sub-optimal solutions quickly. The remainder of this paper is organized as follows: The next part is the literature review, followed by the mathematical model, the fourth part shows the design of the proposed algorithm, and the fifth part reports some experimental results. This paper ends with conclusions and perspectives in the sixth part.

2. Literature review

The studied problem could be regarded as a combination of the VRP model and the pricing model. The literature section is divided into two parts: the research status of VRP with pricing problem, and the related works of ALNS.

2.1. VRP with pricing problem

In the traditional VRP, the transportation cost in each arc is proportional to distance (Abreu et al., 2021; Shi et al., 2020; Lei et al., 2022). Since many variants of VRP are an extension of CVRP, a majority of the model in the VRP area assumed that the transportation prices or costs in each arc are proportional to distance (Niu et al., 2021). However, due to

the flexibility of the pricing strategy could adjust the number of sales and may enhance the potential ability of earnings, some researchers have begun to integrate the pricing strategy into CVRP.

Liu and Chen (2011) noticed that the pricing decision affects the demand decision and then investigated an inventory routing and pricing problem (IRPP) in the supply chain. The paper considered the linear relationship between price and demand. In order to solve the proposed problem, the authors designed a tabu search based approach in which each iteration randomly selects this approach to improve either the inventory routing solution or pricing. Etebari and Dabiri (2016) extended the IRPP by dynamically setting the price based on the period. Similarly, the relationship between price and demand is also assumed to be linear. Furthermore, the paper assumed that the willingness to pay for services may be time-dependent. It is thought that for customers, the value of seasonal goods such as perishables or liquefied gas may change with the seasons. The problem was solved by a hybrid heuristic approach which embedded five stages: initialization, demand generation, demand adjustment, inventory routing, and neighborhood search, in a simulated annealing framework.

Aras et al. (2011) studied a multi-station vehicle routing problem in the context of the reverse supply chain. In the studied problem, the companies purchase goods from dealers to set prices. The purchase is completed if the price is above the dealer's threshold and the company considers the transaction profitable. Two linear model formulas and a tabu search heuristic are developed to solve this problem. Ahmadi-Javid and Ghandali (2014) studied a location-allocation problem with the consideration of price-sensitive demand under compulsory and optional service policies for all customers. A mixed-integer linear programming model is proposed to formulate the problem, and the model is solved by a Lagrange Relaxation algorithm. Ahmadi-Javid and Hoseinpour (2015) further extended the location-allocation model in Ahmadi-Javid and Ghandali (2014) through location inventory and pricing decisions. Recently, Ahmadi-Javid et al. (2018) investigated the location-routing problem by considering discrete prices and corresponding demand levels under selective service policies. To solve the proposed model, the authors have reformulated the model as a set-packing model and developed an exact approach named branch & price to solve the designed instances.

Afsar et al. (2021) made the early attempt to model VRP with taking into account the zone pricing issues. A mixed-integer programming model is proposed, and a branch and price algorithm is designed to solve the model. In order to demonstrate the efficiency of the proposed algorithm, they developed a few groups of benchmark instances and reported the lower bound and the gap for each instance. However, we find that the computational time for the larger instances becomes much large, and currently, there is no heuristic algorithm has been designed to solve the problem.

2.2. ALNS

In the attempt to find a better solution in the neighborhood of the current solution, the local search algorithm is usually easily trapped into a local optimal. To overcome this disadvantage, many new algorithms are introduced, such as simulated annealing, tabu search, and variable neighborhood search. The large neighborhood search (LNS) algorithm and its variants have attracted a significant number of scholars in operations research. LNS, initially proposed by Shaw (1997) with the idea of employing a powerful move operator in a large neighborhood, and experimental results demonstrated the good performance of the naive LNS when solving VRP. After that, many variants have been adapted to solve the combinatorial optimization, especially for the problems related to the routing problem.

Ropke and Pisinger (2006) extended the basic LNS with an adaptive strategy to solve VRP for pickup and delivery with time windows and named the new algorithm as ALNS. In this work, about 350 benchmark instances have been tested, and they reported that the

solutions obtained by ALNS improved more than half of the best-known solutions. Demir et al. (2012) applied ALNS to solve the pollution-routing problem (PRP), which is a typical model in the area of green VRP. Unlike VRP, PRP aims to optimize fuel consumption and green gas emissions, and the model belongs to non-linear mixed-integer programming. Extensive experiments have been performed to solve the challenging PRP, and the results show the good efficiency of the ALNS. Ribeiro and Laporte (2012) designed an ALNS for solving cumulative capacitated vehicle routing problem, and extensive experiments have been performed on the benchmark instances. They have reported comparing the solutions obtained by ALNS and the memetic algorithm. Masson et al. (2013) considered a pickup and delivery VRP with transfers, and the authors designed an efficient ALNS for solving the studied problem. Their ALNS embeds new insertion heuristics which can efficiently insert requests through transfer points. Aksen et al. (2014) developed a tailored ALNS with 11 distinct neighborhood moves to handle a selective and periodic inventory routing problem. Dayarian et al. (2016) studied a multi-period VRP aiming to optimize product collection and delivery from production locations to a set of processing factories over a planning period. The authors proposed an ALNS with certain specifically designed operators and features. Liu et al. (2019) proposed an ALNS for the VRP with time windows and synchronized visits applied in the field of home health care. Sacramento et al. (2019) formulated the VRP with Drones, which is an extension of the classical Flying Sidekick Traveling Salesman Problem (Murray and Chu, 2015). To tackle the model, they have designed ALNS and tested a large number of instances to demonstrate the effectiveness of the ALNS. Chen et al. (2021) investigated the autonomous delivery robots in urban logistics and defined the studied problem as VRP with time windows and delivery robots. Since the model is NP-hard, they have designed ALNS to solve the model. Vincent et al. (2021) has proposed a model to formulate the green mixed fleet VRP with realistic energy consumption and partial recharges, and an ALNS that incorporates the features of the model has been developed to solve the problem. Mezouari et al. (2022) addressed the surgery planning optimization problem with ALNS.

In recent years, researchers have developed hybrid algorithms that integrate ALNS with other optimization techniques to address the combinatorial optimization problems. Gu et al. (2019) proposed a heuristic based on the ALNS and a mathematical programming based operator (MPO) to tackle the commodity-constrained split delivery VRP. In the proposed heuristic, when ALNS finds a new global best solution, MPO is applied to further improve the obtained best solution by reassigning commodities to routes. Akpunar and Akpinar (2021) presented a hybrid metaheuristic algorithm that combines ALNS with variable neighborhood search (VNS) to the capacitated location routing problem. The hybrid metaheuristic adopts the framework of ALNS with VNS embedded, and VNS is implemented when the solutions have not been improved by ALNS for predefined iterations. Liu et al. (2021) developed an efficient metaheuristic that incorporates ALNS with the commercial solver for the multi-period home health care routing and scheduling problem. In the metaheuristic, the commercial solver is called when the best solution is updated during the search of ALNS and solves the simplified model of the problem by fixing some variables. Cai et al. (2022) designed a hybrid algorithm ALNS/TS which integrates ALNS and tabu search (TS) to handle the electric vehicle relocation problem. At each iteration of ALNS/TS, TS is first adopted to search in the local neighborhoods, then ALNS exploits a large neighborhood to escape from the local optimum. (Abreu and Nagano, 2022) studied the open shop scheduling problem with consideration of sequence-dependent setup times, and a new hybridization of ALNS is specially designed to solve the model. Amiri et al. (2023) investigated a bi-objective optimization model for green VRP with taking into account greenhouse gas emissions. ALNS is developed to solve the instances generated based on real-world locations in Canada. Rolim et al. (2023) studied a just-in-time scheduling problem and designed an ALNS to solve the model. Ji et al. (2023) formulated a many-to-many VRP considering the

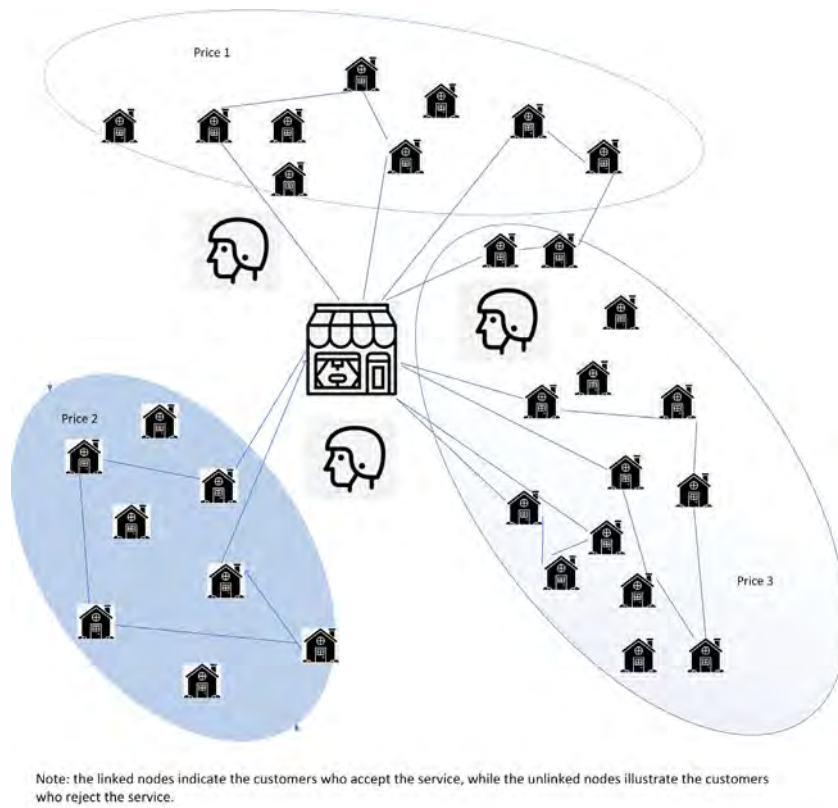


Fig. 2. An example of the studied problem.

constraints of cross-docking and 2-D loading. Two ALNS-based heuristic methods have been proposed to solve the model.

The above literature analysis reveals that the basic CVRP model, as well as being extended into multiple models, is applied in different scenarios. For practical problems, especially community delivery, the community uniform pricing problem is relatively less studied. When designing algorithms for this problem, the number of customers is fixed in traditional heuristics. In contrast, for VRP considering pricing problems, the number of serviced customers is constantly adjusted, which requires us to develop new operators to describe this in the evolutionary computation.

To summarize, despite the significant application of integrating the CVRP and pricing strategy, only a few works have been involved with VRPZ. So far, some exact algorithms have been proposed for the VRP with zone pricing, but there is no heuristic algorithm has been developed to solve VRPZ. Due to the tremendous computational challenges, neighborhood search-based methods have great potential as competitive approaches for real-life scale problems. Besides, the successful application of ALNS and ALNS-based algorithm in solving the routing problem can be observed. Motivated by the above analysis, we develop an efficient and effective hybrid ALNS algorithm to solve the VRPZ. The hybrid algorithm embeds local search into the framework of ALNS using a new mode, and we compare its performances with existing algorithms in the literature.

3. Problem statement

3.1. Assumptions

The problem studied in this paper could be regarded as a vehicle routing problem with taking into account pricing decision making, and the problem is depicted in Fig. 2.

In the problem, the following assumptions can be summarized:

- (1) Geographically distributed customers are pre-divided into multiple zones, and customers in the same zone follow the same price.
- (2) Each customer has a psychologically expected threshold price; if the transportation price is higher than the threshold value of the customer, the service will be rejected.
- (3) The vehicles considered in this study are homogeneous.
- (4) The threshold value of price for each customer is known in advance.
- (5) Each customer's demand could be transported at most in one vehicle.

The problem is defined based on the directed completed graph (V, A) , where V represents all the vertices and A denotes all the arcs on the graph. Specifically, V is composed of all the customers and depots (V_l indicates the customers in zone l), and A comprises all the arcs between the vertices.

The proposed mixed-integer programming model can be seen in Afsar et al. (2021) (we also show the model in the section of the appendix). Since the studied problem is an extension of classical CVRP, which is a typical NP-hard problem (Lenstra and Kan, 1981), there is no doubt that the problem is also an NP-hard problem. In Afsar et al. (2021), the authors has proposed the following lemma, which lays a crucial fundamental of our proposed algorithm.

Lemma. Let th_i be the i th customer's threshold value of the price, and P is a set composed by th_i . Afsar et al. (2021) states that, in the optimal solution, for any zone l , the zone price p_l must belong to $\{th_i : i \in V_l\}$, and the optimal price $p_{best} \in P$.

Proof. Consider that if optimal p_{best} does not belongs to the set P , and R is the optimal route, the best objective value $f(R, p_{best})$ could be written as follows.

- (1) If $p_{best} < \inf P$, we find $f(R, p_{best}) < f(R, \min P)$. This contradicts the hypothesis.
- (2) If $p_{best} > \sup P$, we find $f(R, p_{best}) < f(R, \sup P)$. This contradicts the hypothesis.

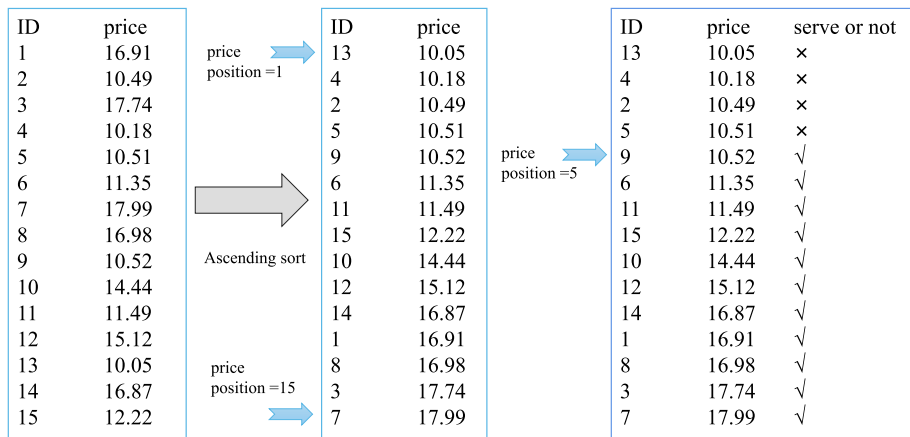


Fig. 3. The demonstration of price position.

(3) If $\inf P < p_{best} < \sup P$, p_{best} must fall into a interval $p_{k-1} < p_{best} < p_k$, we find $f(R, p_{best}) < f(R, p_k)$. This contradicts the hypothesis.

To sum up, we confirm that the optimal solution does not hold in all the above three scenarios. So we have $p_{best} \in P$. ■

Within this conclusion, we have introduced a new concept named price position and this idea will be involved in the meta-heuristic algorithm for solving the problem. We give an example in Fig. 3 to explain the price position. First, 15 customers and their price thresholds are present, then the customers are sorted in ascending order in terms of the threshold value. The price position becomes larger when a higher threshold price of patients is selected as the zone price. In this example, price position = 1 when zone price = 10.05, and price position = 15 when zone price = 17.99. Finally, suppose that the current price position is 5, i.e., zone price is 10.52, then four patients whose threshold values are less than the zone price will refuse the home delivery.

4. Solution approach

In a neighborhood search algorithm, if the neighborhood range is too small, it is challenging to get out of being stuck in a local optimum, even with meta-heuristic techniques such as simulated annealing or tabu search. The large neighborhood search (LNS) algorithm makes up for this deficiency as this algorithm allows multiple neighborhoods to be explored in one search step, and the exploring range in the solution space can be greatly improved. ALNS improves LNS in two main ways: (1) ALNS develops several removals and insertion operators for destroying and repairing solutions, then selects one based on the statistics collected in terms of the heuristic’s historical performance during the search. However, LNS only consists of one pair of removal and insertion method. (2) ALNS uses a strategy similar to simulated annealing (SA) to determine whether the new solution should be accepted as the new input of the next iteration. In contrast, LNS only accepts the new solution that improves the best solution.

The pseudo-code and flow chart of hybrid ALNS are presented in Algorithm 1 and Fig. 4, respectively. The algorithm starts from an initial solution S_0 , and the removal, insertion, and local search operators are predefined. At the beginning of the algorithm, the best solution so far S_{best} and current solution $S_{current}$ equal S_0 . ω^r , ω^i , and ω^l represent the weight sets of removal, insertion, local search operators, respectively. It is worth noting that initial weights for all the operators are the same and equal to 1. Lines 3–16 execute the core part of the proposed hybrid ALNS to improve the solution iteratively until the stop condition is reached. Lines 6–9 are applied to yield the new solution S_{new} . First, a randomly selected price vary operator (see Section 4.5) is applied to deal with $S_{current}$ and generate the temporary solution S_{temp} . Then, the algorithm selects a removal operator to remove some

Algorithm 1: Overview of hybrid ALNS

```

1 Input: an initial solution  $S_0$ ; removal/ insertion/ local search operators set  $\Omega^r / \Omega^i / \Omega^l$ 
2  $S_{best} = S_0$ ;  $S_{current} = S_0$ ;  $\omega^r = (1, \dots, 1)$ ;  $\omega^i = (1, \dots, 1)$ ;  $\omega^l = (1, \dots, 1)$ 
3 while stop condition is not met do
4   Randomly select a price vary operator  $price^*$ 
5   Select a removal operator  $\sigma^- \in \Omega^r$ , an insertion operator  $\sigma^+ \in \Omega^i$  and a local search operator  $localsearch \in \Omega^l$  using the values of  $\omega^r$ ,  $\omega^i$ , and  $\omega^l$ ;
6    $S_{temp} \leftarrow price^*(S_{current})$ ;
7    $S_{temp} \leftarrow \sigma^-(S_{temp})$ ;
8    $S_{temp} \leftarrow localsearch(S_{temp})$ 
9    $S_{new} \leftarrow \sigma^+(S_{temp})$ ;
10  if  $accept(S_{new}, S_{current})$  then
11    |  $S_{current} = S_{new}$ 
12  end
13  if  $f(S_{new}) > f(S_{best})$  then
14    |  $S_{best} = S_{new}$ 
15  end
16  update  $\omega^r$ ,  $\omega^i$ , and  $\omega^l$ ;
17 end
18 Output:  $S_{best}$ 

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customers from S_{temp} . After that, local search is utilized to improve the partially broken solution S_{temp} . Finally, the selected insertion operator inserts the removed customers into the S_{temp} , and S_{new} is obtained. Lines 10–12 accept the S_{new} using the SA criterion, and S_{new} replaces the $S_{current}$ if it performs better (line 13–15). Line 16 updates ω^r , ω^i , and ω^l using adaptive strategies (mentioned in Section 4.6). The algorithms finally outputs S_{best} in line 18.

4.1. Initial solution

As proofed by many works, most heuristic methods are sensitive to the initial solution; therefore, the procedures for generating a high-quality initial solution are critical in the hybrid ALNS. Before generating the initial solution, we assume that the initial price proposed by the transportation company in each zone equals the lowest threshold price of the customer in this zone. That is, all the customers accept the delivery service and will be involved in the initial solution.

In this study, the greedy insertion (see Section 4.4 for the insertion operators) is used to construct the initial solution. First, m (m is large enough to ensure all the customers can be inserted into the routes feasibly) empty routes that only contains the starting and ending points

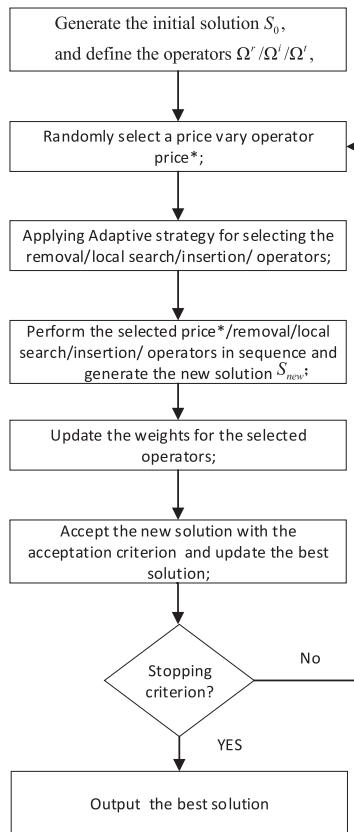


Fig. 4. The flow chart of the hybrid ALNS proposed in this study.

of the vehicle is yielded, and then the customers who accept the home delivery are inserted into the routes using the greedy operator.

4.2. Removal operators

Destruction operations often damage current solution structures, which are beneficial to escape from the local optima. In the existing VRP-related literature, the common principles of the requirement removal are as follows: (1) Randomly select the removed part to ensure the diversity of the search; (2) Remove the worst-performing part, i.e., the part that incurs the high objective value; (3) Select similar units for dismantling, for example, select the points with similar spatial location, quantity, and time window. Based on these three principles, this paper adopts following four removal operators, and each removal operator stops until q customers have been removed.

Related removal: This is a classic removal operation that picks two customers serviced with a high relatedness and splits them. Destroying such a pair of highly related customers may generate new solutions, thus searching for a larger solution space and possibly generating better solutions. The relatedness of related removal in this study denotes the geographical proximity of two customers, and we suppose that the relatedness $R_{i,j}$ of customers i, j equals the customers' distance $d_{i,j}$. The main steps of related removal are presented in Algorithm 2, and Fig. 5 exhibits an example of the related removal.

Zone removal: This operator is a newly devised operator based on the zone feature of the studied problem. Zone removal follows a similar principle with related removal, while it removes the customers in the same zone (the zone is randomly selected before executing the zone removal).

Worst removal: This operator removes the customer that causes high objective value. Let r be a route: $depot \rightarrow v_1 \rightarrow v_2 \rightarrow \dots \rightarrow v_n \rightarrow depot$ and $h(r)$ be the objective value of route r . Let \tilde{r}_i be the

route without node v_i and $\Delta_i = h(r) - h(\tilde{r}_i)$. Therefore, the customer that corresponds high Δ_i should be removed. The procedure of worst removal can be referred to [Ropke and Pisinger \(2006\)](#). Note that the objective value in this paper equals total revenue minus total travel costs. Instead of respectively computing the objective values of $h(r)$ and $h(\tilde{r}_i)$ to obtain Δ_i , we can use the decrement of the travel costs minus the decrement of the revenue to calculate Δ_i quickly. The main steps of related removal are presented in Algorithm 3.

Algorithm 2: The main steps for related removal

```

1 Input: solution S, random parameter p1, set H, empty set Ω;
2 Randomly remove a customer i from S to Ω;
3 while |Ω| < q do
4   Randomly select a customer i from Ω;
5   Record R_{i,j} of i with any customer j in S;
6   Let H contains all customers in S but not in Ω;
7   Sort H in ascending order in terms of R_{i,j};
8   Remove customer H[y^{p1}|H|] from H and insert it into Ω,
   y ∈ [0, 1) is a random number;
9 end
10 Output: S.
    
```

Algorithm 3: The main steps for worst removal

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1 Input: solution S, random parameter p2, set H;
2 for 1 to q do
3   Compute Δ_i for each customer i in S;
4   Let set H contains all customers in S;
5   Sort H in ascending order in terms of Δ_i;
6   Remove customer H[y^{p2}|H|] from H, y ∈ [0, 1) is a random
   number;
7 end
8 Output: S.
    
```

Random removal: Select several customers randomly and remove them directly in the current solution.

4.3. Local search operators

This paper applies local search to improve the partially broken solution generated by the removal operators. Local search explores the neighborhood structures of the candidate solution and tries to find a better solution. Note that ALNS can explore broad and complicated solution spaces; here we just define three classic operators for our local search.

0-1 relocate: Randomly select and remove one customer from the solution, then insert this customer back into the solution. Note that the customer should not be inserted into the place from which it is removed.

1-1 exchange: Randomly select two customers and then exchange their positions.

2-opt: Randomly select two customers in one route, and then reverse the route part composed by these two customers.

2-opt*: Randomly select two links in two distinct routes, and then exchange these two links with each other to compose two new routes.

When the local search is called, the candidate solution is moved to its neighborhood based on the selected operator. Our local search terminates when the local optima is found. More specifically, only feasible solutions are allowed during the local search process. The local search operators are shown with examples in [Figs. 6](#).

4.4. Insertion operators

In this study, we employ two insertion operators to repair the destroyed solution. Each insertion operator starts with a partially broken solution and q customers that need to be re-inserted, then a customer, which has been removed before, is inserted into the current solution

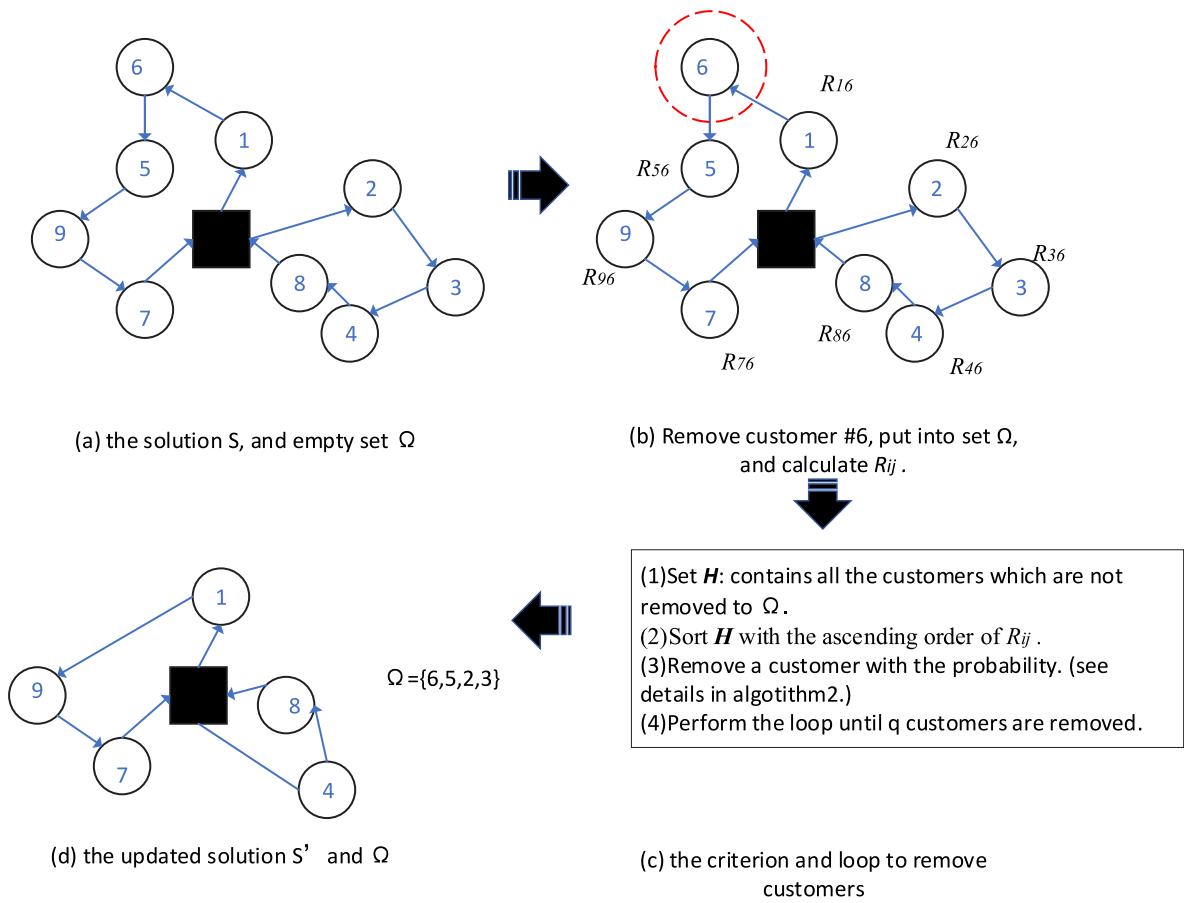


Fig. 5. An example for related removal.

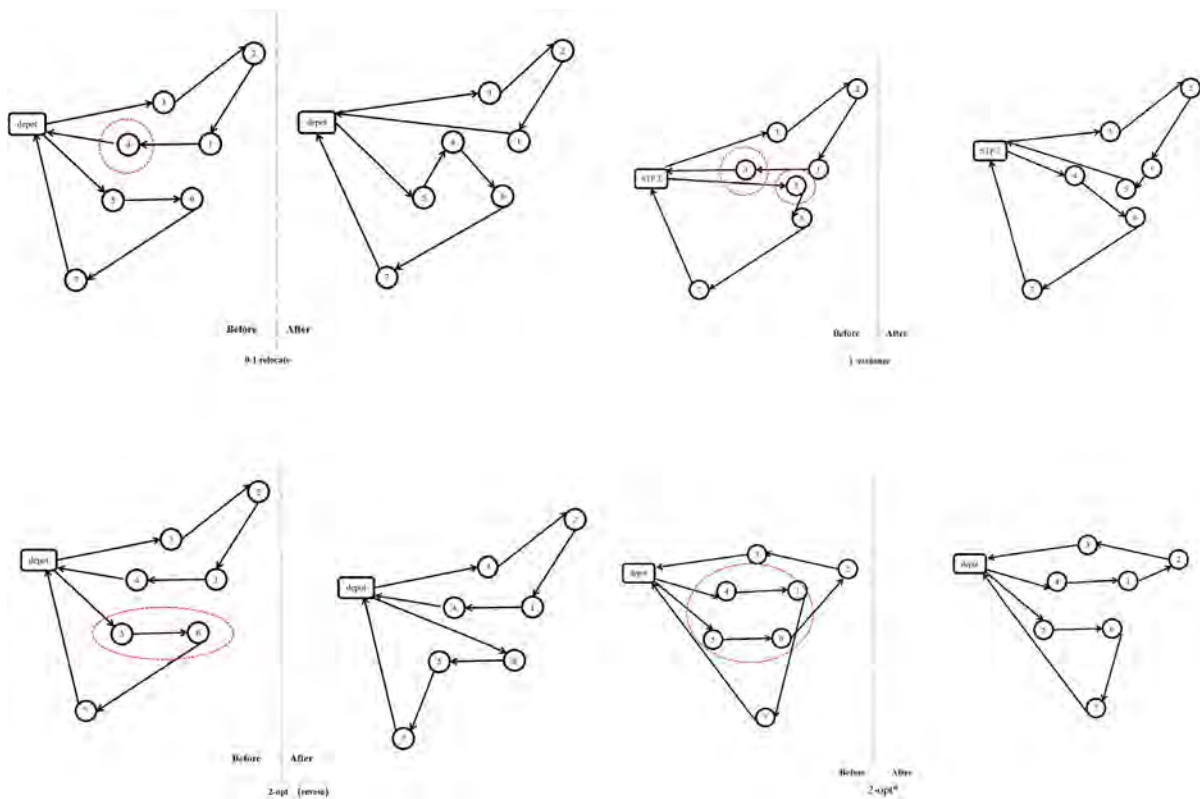


Fig. 6. Examples for the local search operators.

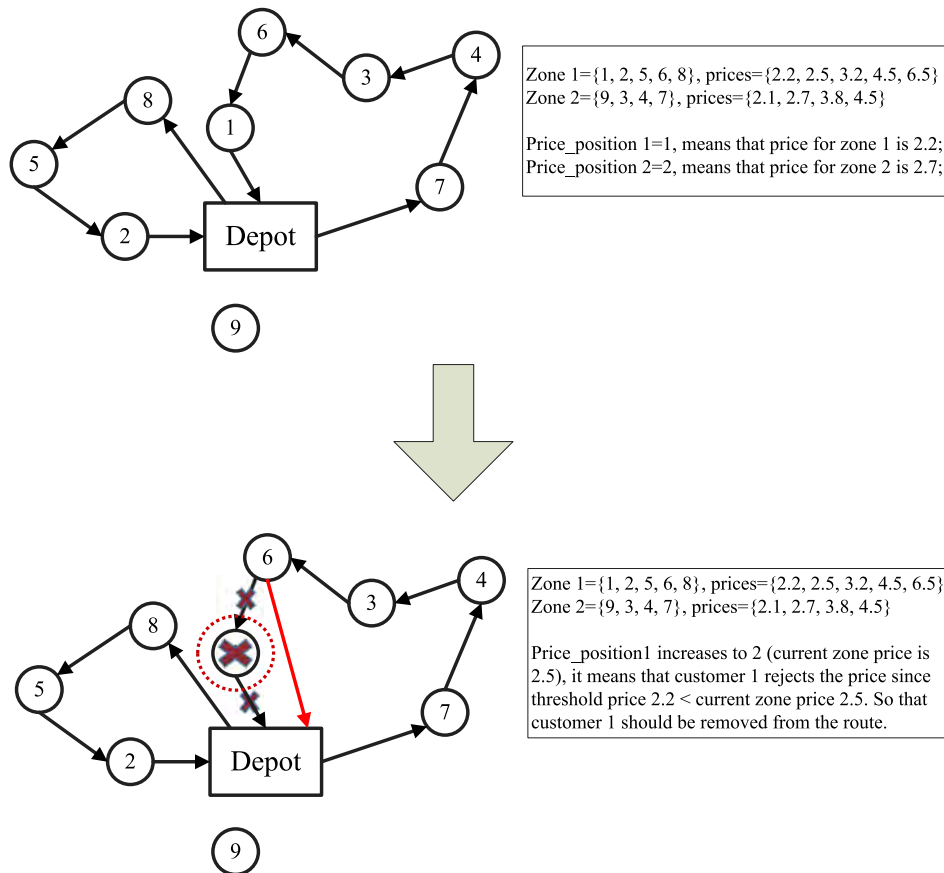


Fig. 7. An example of price ++.

at each iteration and stops when all the removed customers have been inserted.

Greedy insertion: This operator comes from the idea of the best insertion at each iteration. Let us take route $r : depot \rightarrow v_1 \rightarrow v_2 \rightarrow \dots \rightarrow v_n \rightarrow depot$ as an example, and there are at most $n + 1$ feasible positions that can be inserted for each removed customer. Suppose that Δ_{ij} indicates objective change when customer i is inserted into the position j . Then, the customer will be inserted into the position when $\arg \min \Delta_{ij}$ is satisfied. More specially, when position j is not available for customer i , Δ_{ij} is set as $+\infty$.

Regret insertion: This operator inserts the customers into the most regretful place of the solution. Let δ_i indicates the lowest change of the objective value for the insertion of removed customer i , and the best insertion position is located at j . The second lowest change of the objective value is defined as δ'_i , and the corresponding insertion position is j' . Then, $rv_i = \delta'_i - \delta_i$ represents the regret value for the insertion of customer i . For each iteration, regret insertion thus inserts the customer that maximizes the rv_i at its best insertion position.

More specifically, in order to accelerate the process of insertion operators, the change of the objective value can be represented by the difference between the decrement of revenue and the decrement of travel costs.

4.5. Price vary operators

Since the price in each zone is a critical decision variable in our model, the price change could be regarded as the hands of a clock that can swing. One of the critical contributions in this study is that we have developed two price vary operators: namely the *price++* and *price--*, for generating a new solution from the current solution. The detailed procedures for the two operators are presented in Algorithms 4 and 5.

4.5.1. Price++

In this operator, the price position increases with one position, and one customer whose threshold price is lower than the current zone price will be deleted from the route. We give an example in Fig. 7 to demonstrate the *price++*. In Fig. 7, nine customer are distributed into two zones. The price threshold of each customer and current price position of each zone is presented. When the *price++* is applied, supposed that the price position of zone 1 is increased from 1 to 2, then customer 1 whose price threshold is less than the current zone price will be removed. In particular, *price++* will not be applied to the zone whose price position reaches to the maximum value. For instance, when price_position 1 in Fig. 7 equals 5, it is impossible to increase the value of price_position 1.

Algorithm 4: The main procedures for *price++*

- 1 Input: a solution S , including the route information $S.routes$, the price information for all the zones $S.prices$, and the price position information for all the zones $S.pricesPos$;
- 2 $S_{new} \leftarrow S$, and randomly generate a zone ID $l \in Z$;
- 3 $S_{new}.prices[l] \leftarrow S.pricesPos[l] + 1$.
- 4 Calculate the corresponding customer V^* according to $S_{new}.prices[l]$;
- 5 Traverse the $S_{new}.routes$ and find the customer V^* and delete it.
- 6 Update S_{new} .
- 7 Output: S_{new} .

4.5.2. Price--

In this operator, the price for a certain zone will be improved in one position, such that a customer, whose threshold value is not less than current zone price will be inserted into the route. For example, in Fig. 8, when price position of zone 2 is decreased from 2 to 1, customer

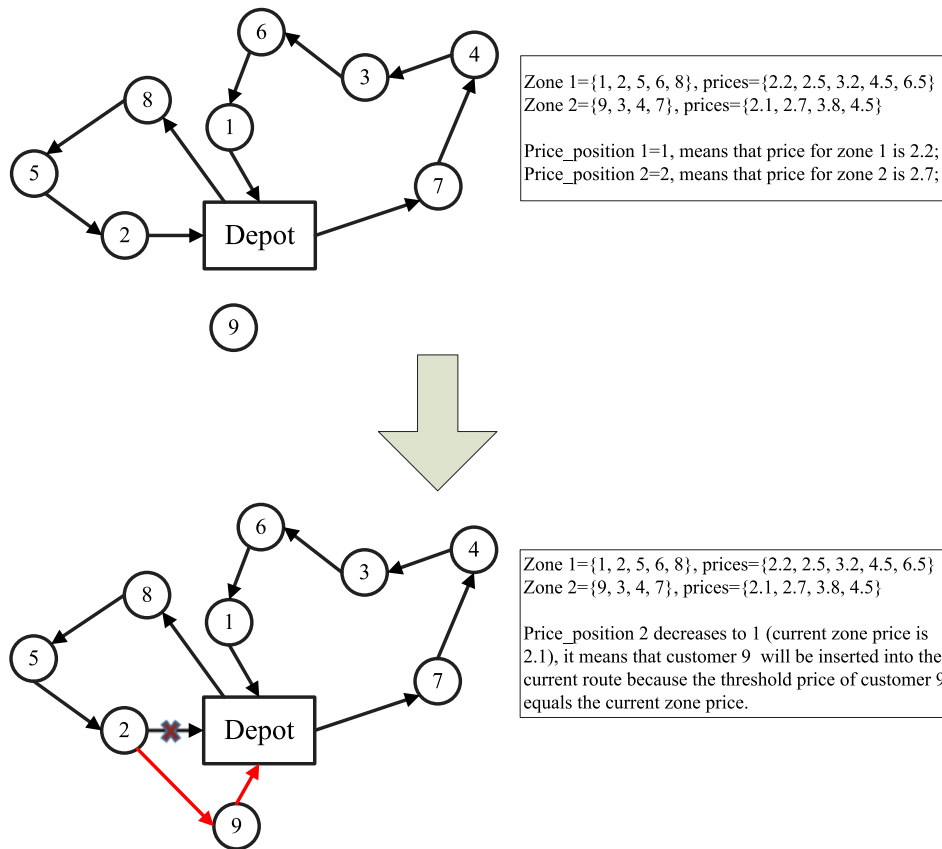


Fig. 8. An example of price --.

9 can be inserted into the route. Similarly, price-- will not be applied to the zone whose price position equals 1.

Algorithm 5: The main procedures for price --

- 1 Input: a solution S , including the route information $S.routes$, the price information for all the zones $S.prices$, and the price position information for all the zones $S.pricesPos$;
- 2 $S_{new} \leftarrow S$, and randomly generate a zone ID $l \in Z$;
- 3 $S_{new}.prices[l] \leftarrow S.pricesPos[l] - 1$.
- 4 Calculate the corresponding customer V^* according to $S_{new}.prices[l]$;
- 5 Insert V^* into $S_{new}.routes$ with the greedy insertion operator.
- 6 Update S_{new} .
- 7 Output: S_{new} .

4.5.3. Unnecessary price variation

In order to accelerate the search process, we give a judging procedure to exclude the unnecessary price variation which indicates that the price operator becomes impossible to improve the global best solution. Note that the studied problem aims to maximize the difference between the revenue and the travel cost, and the travel cost of one solution is always greater than 0, thus the solution whose revenue is less than $f(S_{best})$ (i.e., the objective value of the best solution obtained so far) cannot be optimized to improve the S_{best} . We define an unnecessary price variation such that after the price variation the revenue of the solution is less than $f(S_{best})$. If one price operator has been proved to be an unnecessary variation on a zone, then it can be tried to perform this price operator on other zones or perform another price operator until a necessary variation has been found.

Let us take the solution in Fig. 7 as an example. Before using price++, the revenue of the solution is $2.2 \times 5 + 2.7 \times 3 = 19.1$. After using price++, the revenue of the solution is $2.5 \times 4 + 2.7 \times 3 = 18.1$.

Suppose that $f(S_{best})$ equals 18.5, thus it is futile to utilize price++ to increase the price position of zone 1. In this case, we can choose to perform the price++ on zone 2 or perform the price--.

4.6. Adaptive strategy

As described before, a lot of operators for removal, local search, and insertion have been introduced, and the operators are selected at each iteration based on their weights. In ALNS, the weights are adaptively updated using statistics from earlier iterations. The adaptive strategy described in this section follows the principle of Ropke and Pisinger (2006).

Let us divide the entire hybrid ALNS into a number of segments, and each segment contains 100 iterations. Each operator is associated with a score of 0 at the beginning of each segment. During the segment, if one operator participates in generating the new solution S_{new} , then the score of this operator is increased by σ_1 , σ_2 , and σ_3 according to the quality of S_{new} . σ_1 , σ_2 , and σ_3 respectively correspond to the situation when S_{new} is better than S_{best} , better than $S_{current}$, or worse than $S_{current}$ but still be accepted. Let η_i and θ_i respectively be the total score and usage times of operator i during the segment, and μ represents the reaction rate to the weights in terms of the operator's performance. The weight of operator i applied in segment $j+1$ can be adjusted as follows:

$$w_i^{j+1} = (1 - \mu)w_i^j + \mu \frac{\eta_i}{\theta_i} \quad (1)$$

4.7. Acceptance and stopping criterion

In hybrid ALNS, SA scheme is adopted to determine whether the generated solution S_{new} will be accepted or rejected. We first define an initial temperature T_0 at the start of hybrid ALNS. At each iteration, S_{new} is accepted with a probability calculated by $e^{-(f(S_{new})-f(S_{current}))/T}$. T represents the current temperature and equals T_0 at the beginning.

A cooling factor $\beta \in (0, 1)$ is applied to decrease T such that $T = T \times \beta$. The initial temperature T_0 greatly affecting the performance of SA, and its value is normally positively correlated with $f(S_0)$ (the objective value of the initial solution) (Afifi et al., 2016). Consider that the objective function in this paper focuses on the difference between the total revenue and the total cost, $f(S_0)$ may be negative. Hence, we employ the absolute value of $f(S_0)$ as the reference value to generate T_0 . The algorithm in this paper stops when the maximum number of iterations without improving the solution is reached.

4.8. Other algorithms

To highlight the performance of the proposed hybrid ALNS in this paper, we make a comparison among the solutions obtained by the other heuristic algorithms and hybrid ALNS. Specifically, pure ALNS, variable neighborhood search (VNS), and simulated annealing (SA) are also employed to solve the problems with the same instances, respectively.

Pure ALNS shares the same structure with hybrid ALNS, while it do not include the local search procedure. Variable Neighborhood search (VNS), regarded as an improved local search algorithm, has been widely used in solving combinatorial optimization problems due to its good performance. This algorithm is mainly derived from the idea that the local optimal solution of a single neighborhood structure is not necessarily the local optimal solution of another neighborhood structure, and the global optimal solution is the local optimal solution of all possible neighborhoods. VNS explores new solutions from the solution space according to the neighborhood structure formed by different searching moves and achieves a good trade-off between intensification and diversification.

Algorithm 6 provides the detailed procedures of the VNS, which is as well as starts from a randomly generated initial solution S_0 (see details in Section 4.1). In lines 1–2, the initial solution and neighborhood are given in advance, and the current best solution S_{best} is initialized as S_0 . The rest lines show the evolution process of S_{best} . As crucial procedures of VNS, lines 6–7 illustrate the process of generating new solutions. VNS finally stops when the terminal criteria are reached, and the algorithm returns the best solution. In our VNS, we use four neighborhood structures defined in Section 4.3 for the shaking and local search procedures. Hence, $k_{max} = 4$, and the executed order of the operators is random at each iteration.

Algorithm 6: The pseudo code of VNS

```

1 Input: initial solution  $S_0$  (see details in Section 4.1);
   neighborhood  $N_k, k = 1, 2, \dots, k_{max}$ , and  $k_{max} = 4$ ;
2  $S_{best} \leftarrow S_0$ ;  $\triangleright$  Let the current best solution be the initial solution.
3 while stopping criteria are not reached do
4    $k \leftarrow 1$ ;  $\triangleright$  Let  $k$  equal to 1
5   while  $k \leq k_{max}$  do
6      $S' \leftarrow Shaking(N_k(S_{best}))$ ;  $\triangleright$  Randomly generate  $S'$  from
        $k^{th}$  neighborhood of  $S_{best}$ 
7      $S'' \leftarrow LocalSearch(S')$ ;  $\triangleright$  A local optimal  $S''$  is obtained
       by local search operators
8     if  $f(S'') > f(S_{best})$  then
9        $S_{best} \leftarrow S''$ ;  $\triangleright$  update  $S_{best}$ 
10       $k \leftarrow 1$ ;  $\triangleright$  Reset  $k$ 
11     else
12       $k \leftarrow k + 1$ ;  $\triangleright$  Switch to another neighborhood
13     end
14   end
15 end
16 Return:  $S_{best}$ .

```

SA was originally proposed by Kirkpatrick, and the idea is derived from the solid annealing principle. That is, when the temperature of the solid is very high, the internal energy is relatively large, and the internal particles of the solid are in a fast disorderly motion. While

the temperature of the solid slowly decreases, the internal energy of the solid decreases and the particles slowly tend to move in an orderly manner. Finally, when the solid is at a low temperature (for example, at room temperature), the internal energy is minimized and the particles are in the most stable state. SA simulates the annealing process in the form of an algorithm to solve combinatorial optimization problems and many experimental results have shown that SA has a good performance. SA starts from a generated initial solution with a relatively high temperature, called the initial temperature. With the continuous decrease of temperature, the solution in the algorithm tends to be stable, but the stable solution may be a local optimal solution. Therefore, the SA attempts to jump out of the local optimal solution with a special strategy to accept some weak solutions. That is, in the neighborhood search of SA, SA can not only accept the better solution, but also accept the worse solution with a certain probability involving the Boltzmann constant. This is conducive to the diversity of solutions to possibly get rid of local optima. The pseudo of the SA is shown in Algorithm 7.

Algorithm 7: Framework of the proposed SA

```

1 Input all the initial temperature:  $T_0$ , final terminal temperature:
    $T_F$ , and cooling rate:  $\zeta$ ;
2  $T \leftarrow T_0$ ; // initialize the current temperature
3  $S_0 \leftarrow Initialization()$ ; // generate the initial
   solution
4  $S_{current} \leftarrow S_0$ ;
5  $S_{best} \leftarrow S_0$ ; //  $S_{best}$  is the best solution found so
   far;
6 while ( $T \geq T_F$ ) do
7    $S_{new} \leftarrow Neighborhood\ operation(S_{current})$ ;
8    $\Delta \leftarrow f(S_{current}) - f(S_{new})$ ;
9   if  $\Delta < 0$  then
10     $S_{current} \leftarrow S_{new}$ ;
11   else
12     $r \leftarrow Random(0, 1)$ ;
13    if  $r < exp(-\Delta/T)$  then
14       $S_{current} \leftarrow S_{new}$ ;
15    end
16   end
17   if  $f(S_{new}) > f(S_{best})$  then
18      $S_{best} \leftarrow S_{new}$ ; // update the current best
     solution
19   end
20    $T \leftarrow \zeta * T$ ; // update the current temperature
21 end
22 Output the best solution  $S^*$ ;

```

5. Computational experiments

This section tests the proposed hybrid ALNS on the instances designed by Afzar et al. (2021), in which three data sets have been created. The first data set involves the instances with 35 customers and 3 zones, and the second data set contains the instances with 50 customers and 3 zones, while the last one consists of the instances with 50 customers and 5 zones. Besides, the authors further classified the instances in each data set into four categories according to the different threshold profiles of customers. Customers in low/high threshold instances are not willing/willing to pay much for the home delivery. Customers in medium threshold instances can accept to pay the price slightly above the travel cost from their home to the depot. Finally, random threshold instances include the customers with a mixed profile regarding willingness to pay for home delivery. Therefore, 12 groups of instances are given in total.

The rest of this section first tunes the parameters of the proposed algorithm, and then extensive experiments are conducted on the benchmark instances using hybrid ALNS and two compared algorithms. All the algorithms are coded in C++, and the programs are compiled with

Table 2

The average values of best solutions and execution times for different parameter configurations.

$(P_{t_1}, P_{t_2}, P_{t_3})$	Ave_best	Ave_time	$(P_{t_1}, P_{t_2}, P_{t_3})$	Ave_best	Ave_time
([0.05, 0.15], 1, 50)	812.11	16.24	([0.25, 0.35], 1, 50)	826.96	30.14
([0.05, 0.15], 1, 80)	820.49	22.01	([0.25, 0.35], 1, 80)	831.42	31.26
([0.05, 0.15], 1, 100)	821.12	28.91	([0.25, 0.35], 1, 100)	830.15	38.47
([0.05, 0.15], 5, 50)	820.2	20.13	([0.25, 0.35], 5, 50)	832.93	36.52
([0.05, 0.15], 5, 80)	825.17	24.8	([0.25, 0.35], 5, 80)	833.28	40.19
([0.05, 0.15], 5, 100)	825.85	33.43	([0.25, 0.35], 5, 100)	834.66	52.73
([0.05, 0.15], 10, 50)	821.49	19.82	([0.25, 0.35], 10, 50)	829.01	31.42
([0.05, 0.15], 10, 80)	827.38	25.77	([0.25, 0.35], 10, 80)	833.16	33.72
([0.05, 0.15], 10, 100)	828.92	39.33	([0.25, 0.35], 10, 100)	832.45	49.9
([0.15, 0.25], 1, 50)	820.73	23.36	([0.35, 0.45], 1, 50)	823.33	35.57
([0.15, 0.25], 1, 80)	822.39	26.63	([0.35, 0.45], 1, 80)	826.69	44.81
([0.15, 0.25], 1, 100)	823.18	34.21	([0.35, 0.45], 1, 100)	831.25	56.62
([0.15, 0.25], 5, 50)	829.27	30.15	([0.35, 0.45], 5, 50)	828.1	39.69
([0.15, 0.25], 5, 80)	835.68	33.38	([0.35, 0.45], 5, 80)	833.34	45.59
([0.15, 0.25], 5, 100)	836.11	45.13	([0.35, 0.45], 5, 100)	834.79	62.91
([0.15, 0.25], 10, 50)	825.44	24.21	([0.35, 0.45], 10, 50)	824.91	35.28
([0.15, 0.25], 10, 80)	830.12	29.96	([0.35, 0.45], 10, 80)	826.76	44.86
([0.15, 0.25], 10, 100)	833.9	42.63	([0.35, 0.45], 10, 100)	830.88	60.66

Table 3

The values of all parameters of hybrid ALNS.

Parameter	Symbol	Value
Number of nodes removed	q	$[0.15 \times V , 0.25 \times V]$
Randomness parameters in related and worst removal	p_1, p_2	3,3
Parameters in the adaptive strategy	$\sigma_1, \sigma_2, \sigma_3, \mu$	9, 33, 13, 0.1
Initial temperature	T_0	$ f(S_0) \times 5$
Cooling factor	β	0.99975
Maximum iterations without improving the solution	Max_ite	$ V \times Num_zone \times 80$

visual studio 2019 in a Windows environment. All the experiments are tested on a processor with 8 GB RAM and Intel Core i7-8750H CPU, 2.20 GHz.

5.1. Parameters tuning

The parameters of hybrid ALNS are tuned using the design of experiments (DoE). This method first defines a set of possible configurations of parameters, and then each configuration of parameters is tested on a number of instances. Finally, the configuration that allows the algorithm to achieve a good trade-off between the execution time and the solution quality is selected. In the preliminary experiments, we find that three parameters, such as the number of removed nodes q , initial temperature T_0 , and maximum iterations without improving the solution Max_ite , have significant effects on the quality of the proposed hybrid ALNS. Hence, we mainly apply DoE method to tune these three parameters, whereas the values of other parameters such as parameters in removal heuristics and the adaptive strategy are determined according to the suggestions of the classical ALNS paper (Ropke and Pisinger, 2006).

In this section, we define that $q = P_{t_1} \times |V|$, and $T_0 = P_{t_2} \times |f(S_0)|$, and $Max_ite = P_{t_3} \times Num_zone \times |V|$, where $|V|$, $|f(S_0)|$, Num_zone denote the number of customers in the instances, the absolute value of the initial solution's objective value, and the number of zones in the instances, respectively. It is stated that $P_{t_1} \in \{[0.05, 0.15], [0.15, 0.25], [0.25, 0.35], [0.35, 0.45]\}$, $P_{t_2} \in \{1, 5, 10\}$, and $P_{t_3} \in \{50, 80, 100\}$. Therefore, there are $4 \times 3 \times 3 = 36$ parameter configurations in total. By selecting one instance from each instance group, 12 instances derived from benchmark instances of Afsar et al. (2021) are selected for the experiments of DoE method. The average values of the best solution ("Ave_best") and the execution time ("Ave_time") for each parameter configuration in solving 12 instances are presented in Table 2. The results show that the configurations ([0.15, 0.25], 5, 80) and ([0.15, 0.25], 5, 100) are relatively better in terms of "Ave_best". However, it is found that when the value of P_{t_3} is changed from 80

to 100, the "Ave_best" is slightly improved (from 835.68 to 836.11, 0.05%) with the significant increase (from 33.38 to 45.13, 35.2%) in "Ave_time". Consequently, we select ([0.15, 0.25], 5, 80) as the final parameter configuration for the proposed hybrid ALNS, and the values of all parameters of hybrid ALNS are listed in Table 3.

5.2. Numerical results

This section compares the experimental results obtained by the proposed algorithms with the benchmarks in Afsar et al. (2021). Note that the experimental results reported by Afsar et al. (2021) is the best results found so far, but it does not mean that all solutions are globally optimal. Because in their paper, they present the Gap between the best integer solutions found and the lower bound (see page 8 in Afsar et al. (2021)), and some of the values of gap are greater than 0. For each instance, we run each algorithm ten times. Tables 4–15 present the compared results for all the standard instances. In the tables, the first column indicates the label of each instance. In the column "Benchmarks", the subcolumns "Opt" and "Times" respectively represent the objective value of the best solution and the execution time in seconds. The results are derived from Afsar et al. (2021). In the columns "Hybrid ALNS", "ALNS", and "VNS", the subcolumns "Opt", "Ave", "Worst", indicate the best, average, worst solution obtained in ten runs. "Std" refers to the calculated standard deviation. "Gap" stands for the gap between the "Opt" obtained by each algorithm and that of the benchmark, which equals $(Opt_{Algorithm} - Opt_{Benchmark}) \times 100\% / Opt_{Benchmark}$. Considering that this paper focuses on solving a maximization problem, the value of the gap remains larger than 0 when our solutions are better than the benchmarks. The best solution for such instances is marked by boldface. Specially, in order to conduct a fair comparison of algorithms, we run each algorithm for the same execution time. In the experimental process, we find that hybrid ALNS usually takes longer time to coverage, thus the run time of hybrid ALNS is taken as the executed time for other two algorithms.

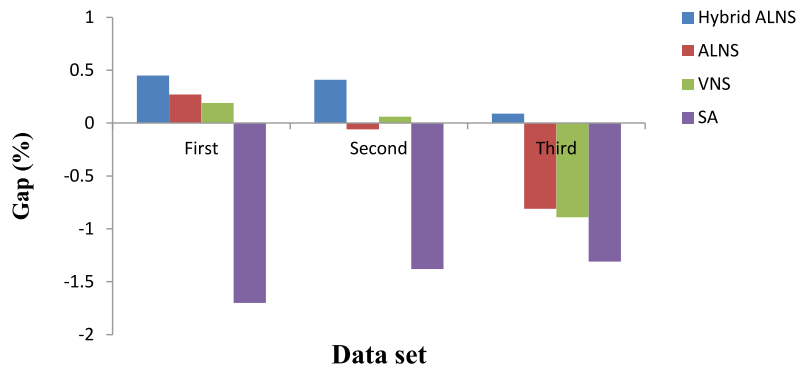


Fig. 9. Compared average gaps in terms of three data sets.

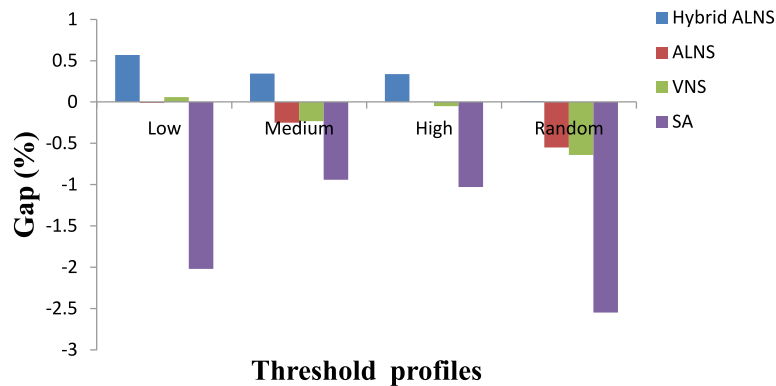


Fig. 10. Compared average gaps in terms of different threshold profiles.

In Tables 4–7, we observe that hybrid ALNS strongly outperforms the method of Afsar et al. (2021) in terms of the first data set since this algorithm finds existing or better benchmarks for almost all the instances in this data set. ALNS and VNS also show competitive performance compared with the method of Afsar et al. (2021), but their results are worse than those of hybrid ALNS. However, the results of SA are not that good in comparison with benchmarks.

Tables 8–15 show that the performance of hybrid ALNS is also efficient for the second and third data sets. Out of total 304 instances in these two data sets, hybrid ALNS obtains 199 (65.46%) best solutions compared with other algorithms and benchmarks. Moreover, hybrid ALNS can significantly improve the benchmarks for some instances; for example, hybrid ALNS improves the benchmarks by 5.13% for the instance *vrpnc07_2* with low threshold in the first data set. Although hybrid ALNS cannot find the existing benchmarks for some instances, the gap between the best solution of hybrid ALNS and the benchmark is always larger than -4% . However, ALNS, VNS, SA are powerless to solve these instances efficiently, as they only find the best solution for 61 (20.07%), 72 (2.68%), 58(19.08%) instances out of total 304 instances.

It is observed from Tables 4–15 that the feasible solutions for some instances (such as the instance *vrpnc11_2* with low threshold profiles, and the instances *vrpnc2_3* and *vrpnc11_3* with random threshold profiles in the first data set.) cannot be found by Afsar et al. (2021) in a reasonable time (10 000 s), and the benchmarks for these instances are marked with “—”. However, the proposed algorithms can solve these instances with short times, and the solution results also further prove the best performance of hybrid ALNS in solving VRPZ.

Table 16 summarizes the average results obtained by the proposed algorithms and benchmarks for 12 instance sets. The first column in Table 16 represents the label of the instance category. For example, (35, 3, low) indicates the instance category with 35 customers, 3 zones and low threshold. According to the average value of “Opt” in Table 16, the

optimal solutions found by hybrid ALNS are better than the benchmarks in terms of 11 instance categories (especially for the instance category (35, 3, low)), and only slightly worse (gap = -0.26%) than those for instances with 50 customers, 5 zones, and random threshold. The optimal solutions found by ALNS and VNS are better than benchmarks for all the instance sets with 35 customers, 3 zones, and partial sets with 50 customers, 3 zones. However, ALNS and VNS are more inefficient than the method of Afsar et al. (2021) for the instance sets with 50 customers, 5 zones. SA is inefficient for all the instance sets.

Table 16 also highlights that hybrid ALNS is more stable than the other two algorithms since the “std” of hybrid ALNS is smaller than those of ALNS and VNS, and the largest “std” is 8.21, which is not that large.

In addition, Table 16 further reveals that hybrid ALNS is more effective than the approach of Afsar et al. (2021) as the run times can be dramatically reduced when hybrid ALNS is applied. For example, the average computational time consumed by hybrid ALNS for the instance set (35, 3, low) is 5.27s, while the corresponding average run time of Afsar et al. (2021) equals 6173.56s, and hybrid ALNS is thus more than 1000 times faster than the method of Afsar et al. (2021) for this instance category. Even if the run time of hybrid ALNS increases with the customers and zones, hybrid ALNS is also nearly 100 times more effective than the benchmark times in the worst case. Moreover, as Afsar et al. (2021) tested their algorithm on a PC with 128 GB RAM and we run our hybrid ALNS on a 8 GB RAM processor, the actual running time gap will be larger if the same hardware and software environments are provided.

Figs. 9–10 record the average gap of the proposed algorithms and benchmarks in terms of different data sets and threshold profiles, respectively. Fig. 9 shows that the benchmarks are improved by hybrid ALNS for all the instance sets, especially for the first data set. ALNS can only find the better benchmarks for the first data set, and VNS improves the benchmarks for the first and second data sets. As can be

Table 15 Compared results for the third data set (instances with 50 customers, 5 zones and random threshold).

Table with 25 columns: Instance, Benchmarks (Opt, Time(s)), Hybrid ALNS (Opt, Ave, Worst, Std, Time(s), Gap(%)), ALNS (Opt, Ave, Worst, Std, Gap(%)), VNS (Opt, Ave, Worst, Std, Gap(%)), and SA (Opt, Ave, Worst, Std, Gap%). Rows list instances from vrpnc01 to vrpnc14_3 with corresponding performance metrics.

Table 16
Compared average results for different instance categories.

Instance	Benchmarks		Hybrid ALNS					ALNS					VNS					SA					
	Opt	Time(s)	Opt	Ave	Worst	Std	Time(s)	Gap(%)	Opt	Ave	Worst	Std	Gap(%)	Opt	Ave	Worst	Std	Gap(%)	Opt	Ave	Worst	Std	Gap(%)
(35, 3, low)	291.8	6173.56	295.51	292.31	289.13	2.1	5.27	0.98	294.15	288.27	281.22	4.06	0.8	293.51	287.46	279.14	4.5	0.58	287.31	257.46	220.48	18.06	-3.00
(35, 3, medium)	521.67	4127.21	523.67	520	515.88	2.48	4.66	0.32	521.81	517.8	512.07	3.24	0.03	521.26	514.07	505.06	5.52	-0.08	518.61	489.74	450.94	17.64	-0.94
(35, 3, high)	1011.2	3568.52	1016.02	1013.37	1010.5	1.92	7.1	0.39	1013.14	1010.82	1007.25	1.93	0.19	1013.6	1006.16	994.83	6.38	0.24	1008.95	982.98	948.49	16.46	-0.40
(35, 3, random)	541.78	2085.33	542.86	540.75	533.61	1.97	4.81	0.12	542.05	537.53	530.93	3.62	0.05	541.78	532.15	521.68	6.46	0	526.02	482.42	436.10	24.62	-2.46
(50, 3, low)	458.1	8124.6	462.4	455.24	447.09	4.77	38.12	0.81	458.15	448.46	436.93	6.23	0.01	459.39	449.81	437.67	6.64	0.28	455.04	421.69	370.35	20.86	-1.40
(50, 3, medium)	783.8	8452.67	788.87	781.5	773.62	4.55	38.08	0.45	782.91	773.29	763.06	6.07	-0.11	784.34	775.29	765.58	4.43	0.07	780.54	745.37	698.56	22.62	-0.97
(50, 3, high)	1473.61	9143.9	1480.01	1472.39	1464.37	4.47	39.51	0.32	1474.26	1464.57	1453.65	6.27	0.04	1475.66	1466.41	1454.76	4.81	0.14	1470.23	1440.30	1392.04	19.47	-0.43
(50, 3, random)	862.34	5056.14	863.74	855.3	845.87	5.31	36.89	0.18	860.72	848.89	835.4	7.83	-0.19	860.13	849.18	837.85	6.43	-0.26	840.88	770.79	694.36	41.00	-2.72
(50, 5, low)	552.25	7872.64	553.7	542.94	533.46	5.81	61.98	0.04	547.64	532.6	517.75	8.39	-0.84	548.58	533.54	519.86	7.79	-0.67	547.43	521.34	483.23	20.61	-1.65
(50, 5, medium)	861.13	8337.33	864.29	854.6	844.47	5.74	63.44	0.3	855.33	842.54	829.3	8.11	-0.68	855.34	841.52	828.46	7.6	-0.68	850.51	828.77	792.81	18.67	-0.91
(50, 5, high)	1642.23	9011.26	1647.1	1637.47	1628.97	5.77	69.35	0.3	1638.58	1628.54	1618.12	6.08	-0.22	1633.53	1620.52	1607.24	7.21	-0.53	1628.79	1607.74	1572.76	18.61	-0.20
(50, 5, random)	924.19	5678.11	920.86	908.56	896.36	8.21	63.26	-0.19	910.42	894.15	879.3	9.13	-1.51	909.01	896.54	881.43	8.44	-1.67	893.94	857.09	804.06	28.99	-2.46

Table 17
The statistic tests for the experimental results.

	Algorithm	Average rank	
Instances with 35 customers and 3 regions	1	2.7818	
	2	1.7152	
	3	2.6788	
	4	2.6606	
	5	4.2364	
	Test	p-value	Comment
	2-1	2.44E-16	Reject H_0
	2-3	7.86E-17	Reject H_0
	2-4	1.02E-14	Reject H_0
	2-5	8.91E-31	Reject H_0
Instances with 50 customers and 3 regions	Algorithm	Average rank	
	1	1.649	
	2	1.7152	
	3	3.2715	
	4	3.0464	
	5	3.7086	
	Test	p-value	Comment
	2-1	3.12E-10	Reject H_0
	2-3	3.39E-20	Reject H_0
	2-4	2.46E-19	Reject H_0
2-5	3.54E-18	Reject H_0	
Instances with 50 customers and 5 regions	Algorithm	Average rank	
	1	2.4056	
	2	1.6294	
	3	3.4755	
	4	3.6014	
	5	3.4825	
	Test	p-value	Comment
	2-1	1.46E-25	Reject H_0
	2-3	7.90E-59	Reject H_0
	2-4	4.21E-53	Reject H_0
2-5	3.27E-57	Reject H_0	

note:1:benchmarks; 2:hybrid ALNS; 3:ALNS; 4:VNS; 5:SA.

seen from Fig. 10, the increase of threshold values (from low to high) decreases the improvement level (the gap from 0.57% to 0.34%) of hybrid ALNS, and hybrid ALNS improves the benchmarks of instances with random threshold profiles the least. In particular, ALNS and VNS can only find better benchmarks for the instances with low threshold profiles. SA is powerless for solving all the sets of instances since the gaps of this algorithm are always less than zero. It is also concluded that the instance set with random threshold profiles is the hardest set to be improved.

5.3. Statistic test

In this paper, a variety of heuristic algorithms are proposed to solve the same instances. In order to further verify the performance of the algorithm, we use statistical analysis to illustrate (Shi et al., 2017). Friedman's test is a non-parametric statistical test proposed by Milton Friedman. The purpose of this approach is to detect processing differences across multiple tests. Considering the specific characteristics of our work, we examine each pair of different methods. Here are two hypotheses.

H_0 : The difference between the solutions obtained by the two methods follows a symmetric distribution around zero;

H_1 : The difference between the solutions obtained by the two methods does not follow a symmetric distribution around zero;

The results are reported in Table 17. We can find the average rank values and the p-values for instances that are divided into three groups. The Average values show that hybrid ALNS performs better than other algorithms tested in the paper. Additionally, the p-values are all less than 0.05, which rejects H_0 and indicates the results have a significant difference.

6. Managerial insights

The model and algorithm studied in this paper have critical management implications in specific business practice. First of all, in the classical routing planning problem model (such as CVRP, VRP with time windows, etc.), it is generally assumed that the transportation price is given (probably not optimal). But in fact, in order to facilitate unified management in these practical problems, such as in the management of ride-sharing services and fast-food delivery services, pricing is often considered along with route planning. The study in this paper provides reference for this management approach. In addition, in the previous studies on the exact algorithm, the calculation time is time-consuming. In practice, the route planning and pricing strategy is a fast-decision-making problem. When the decision maker receives the orders, he or she generally attempts to make a quick optimal decision within a short time (such as within 60 s). The algorithm proposed in this study can quickly give effective and reliable solutions to the designed instances, so the development of hybrid ALNS is essential in practice. This algorithm is also the premise of further developing decision support system in the related industrial management.

7. Conclusions and perspectives

Regarded as an extension model of CVRP, VRPZP has potential applications in practice. Current research on this model mainly focuses on solving it using exact algorithms, which are not too efficient when the problem is large. In this paper, based on this, the ALNS algorithm is designed to solve these arithmetic cases for the benchmark instances in previous studies and the problem's characteristics. A total of 472 instances are tested in this study, and the experimental results show

that the algorithm has good computational accuracy and significantly improves the solution time. One of the limitations of the ALNS is that the selection of the neighborhood operators is still not effective enough for all the instances, we will employ the deep reinforcement learning method in the ALNS in the future to guide the local search operators in exploring better solutions. Additionally, The vehicle routing problem with the pricing strategy in this paper has an actual application in the take-out industry. However, this study does not consider the customer's time window and multiple restaurant departure points. Therefore, in future research, the model will be further extended to consider multiple departure points and customers' time windows (Jie et al., 2022). The results will be simulated based on actual take-out data.

CRedit authorship contribution statement

Yong Shi: Conceptualization, Methodology, Software, Writing – original draft. **Wenheng Liu:** Methodology, Software, Writing – review & editing. **Yanjie Zhou:** Conceptualization, Methodology, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data that has been used is confidential.

Acknowledgments

This work was financially supported by Research Grant from CAST with the Science and Technology Think Tank Program (No. 20220615Z Z07110348) and National Natural Science Foundation of China (NSFC) (No. 72201252). The first author is grateful for the valuable suggestions given by team members during the weekly project discussion meetings during the project implementation of NSFC (Nos. 72074198, 71874165), and National Social Science Fund of China Major Program No. 21AZD074.

Appendix

Sets and parameters:

- Z: set of the zones.
- V₀: set of all the customers.
- O: the unique depot.
- V_i: set of all the customers in the *i*th zone.
- V: set of all the vertices, including all the customers and depots on the defined graph. $V = \cup_{i \in Z} V_i \cup O$;
- q_{*i*}: the demand for customer *i*.
- Q: the capacity of each vehicle.
- \bar{Q} : $Q + \max_{i \in V_0} \{q_i\}$.
- th_{*i*}: the threshold value of price for each customer.
- z(*i*): the zone ID where the customer *i* located in, and obviously z(·) is a function depends on the customers.
- c_{*ij*}: the transportation cost between vertices *j* and *j*.

Decision variables:

- x_{*ijk*}: if vehicle *k* travels from node *i* to node *j*, in which $i \neq j$, x_{*ijk*} = 1. Otherwise, x_{*ijk*} = 0.
- ω_{*i*}: if the customer *i* accepts the price, ω_{*i*} = 1; otherwise ω_{*i*} = 0;
- p_{*l*}: the transportation price in zone *l*
- u_{*i*}: total delivered load when the vehicle leaves node *i*; u_O = 0.

$$\max f(R, P) = \sum_{i \in V_0} w_i \cdot p_{z(i)} - \sum_{i \in V} \sum_{j \in V} c_{ij} \cdot x_{ij} \tag{2}$$

$$w_i \cdot p_{z(i)} \leq th_i, \forall i \in V_0 \tag{3}$$

$$\sum_{i \in V_k} w_i \geq 1 \quad \forall k \in \{1, \dots, |Z|\} \tag{4}$$

$$\sum_{i \in V} x_{ij} = w_j \quad \forall j \in V_0 \tag{5}$$

$$\sum_{i \in V} x_{ij} - \sum_{i \in V} x_{j,i} = 0 \quad \forall j \in V \tag{6}$$

$$u_i - u_j + q_j + \bar{Q} \cdot x_{i,j} \leq \bar{Q} \quad \forall i \in V, \forall j \in V_0 \tag{7}$$

$$u_i \leq w_i \cdot Q \quad \forall i \in V_0 \tag{8}$$

$$w_i \in \{0, 1\} \quad \forall i \in V_0 \tag{9}$$

$$x_{ij} \in \{0, 1\} \quad \forall i, j \in V \tag{10}$$

The objective function (2) maximizes the difference between the total price charged and the travel cost. According to constraints (3), the customer accepts the service if the price proposed for their area is less than or equal to their threshold. Constraints (4) compel the carrier not to completely abandon a geographic area and serve at least one customer. If the customer agrees to the service, the vehicle should visit him, subject to constraints (5)–(6). Constraints (7) are flow equations for the demand. Moreover, the subtours can be eliminated using constraints (7) (Miller–Tucker–Zemlin subtour elimination). The total load delivered by each vehicle is limited by the capacity under constraints (8). Constraints (9)–(10) define the range of the variables.

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