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Cost-sharing contract design between manufacturer and dealership considering the customer low-carbon preferences

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ABSTRACT

Manufacturers and dealerships usually have a long-term cooperation in selling low-carbon products produced by signing a contract without considering the dynamic changes in consumer's low-carbon preferences. Consumers' low-carbon awareness and the low-carbon reputation of those low-carbon products occasionally alternate affected by the market circumstances, which will influence the selling of low-carbon products. This paper focuses on the contract design between a manufacturer and a dealership using Stackelberg differential game models to formulate the long-term contract between a manufacturer and a dealership. To promote the low-carbon reputation of low-carbon products, two cost-sharing contracts, including a one-way cost-sharing contract (OWC) and a two-way cost-sharing contract (TWC), are proposed. The manufacturer in the OWC contract provides a certain amount of subsidies to incentivize the dealership. The manufacturer and the dealership share costs in the TWC contract. Besides, dual-channel models are also analyzed as an extension of the proposed model. Several findings were obtained by solving and analyzing the proposed game models. The manufacturer's emission reduction efforts and the dealership's low-carbon promotion efforts are influenced by dealership's cost coefficient related to the promotion of low-carbon product, low carbon reputation sensitivity coefficient of consumers, influence coefficient of manufacturer's emission reduction efforts on low-carbon reputation and etc. The proposed cost-sharing contracts improve the manufacturer and the dealership's efforts by changing the factors. The OWC contract can achieve Pareto improvement, and the TWC contract can realize the coordination between the manufacturer and the dealership. The optimal conditions for its contract choice are also proposed from the manufacturer's point of view. This paper also provides valuable insights for companies in the supply chain to design suitable contracts to coordinate the supply chain stakeholders' activities, especially in promoting low-carbon efforts in a long-term cooperation.

1. Introduction

Climate and environmental changes have become a worldwide challenge issue for a long time, especially for developing countries, such as, China and India. Various policies and new technologies (Darbha & Pagilla, 2010), including the carbon tax (Tiwari, Wee, Zhou, & Tjoeng, 2021), carbon cap, and trade, are emerging to force enterprises to reduce carbon emissions to address the climate and environmental problems. Many governments and international environmental organizations have been making policies to incentivize manufactures to produce low-carbon products. To create customers' environmental awareness, governments also give customers subsidies for choosing low-carbon products. Such as, in Korea, the US, and China, customers who buy an electric vehicle will get a certain amount of subsidy (Staff, 2020). Consumers' low-carbon preference behavior will cause climate change. However, the relationship between consumer behavior and climate change is complex (Thøgersen, 2021). Nowadays, consumers have a high level of environmental awareness and a stronger preference for choosing a low-carbon product, requiring manufacturers to reduce carbon emissions through research and development or using new equipment to satisfy policy and consumer requirements.

Usually, customers' low-carbon preferences are changing and affected by internal and external factors, which come from the customers themselves and the selling market. Many previous studies stated that customers' low-carbon preference involves the new energy vehicles selling. Here, we give an example to show the customer's low-carbon preference changing for selecting new energy vehicles. Fig. 1 shows the

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[1] CNN: https://edition.cnn.com/interactive/2019/08/business/electric-cars-audi-volkswagen-tesla insideevs.com: https://insideevs.com/news/396714/world-top-10-plugin-automotive-groups-2019/ cleantechnica.com: https://cleantechnica.com/.

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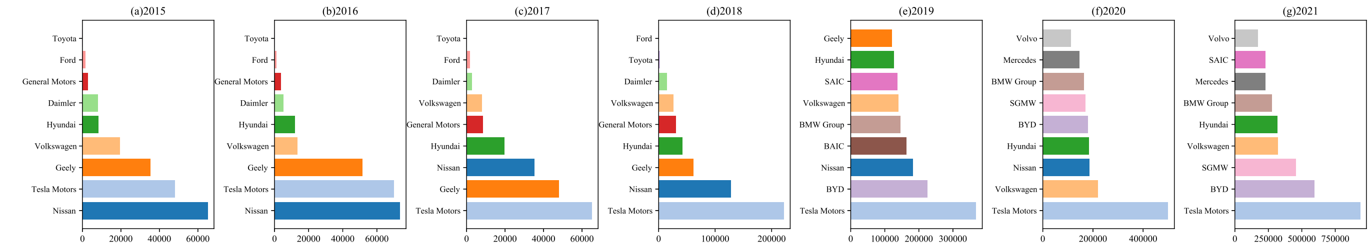


Fig. 1. Top nine electric vehicle automaker according to sale volume from 2015 to 2021.

top nine electric vehicle automaker according to sale volume from 2015 to 2021. The data source of Fig. 1 are provided by CNN, insideevs.com and cleantechnica.com.[1] From Fig. 1, we can found that the selling volume of each brand is changing drastically from 2015 to 2019.

Stimulating consumers to buy low-carbon products integrated with innovative technologies to reduce low-carbon is essential for a manufacturer, especially for automakers producing new energy vehicles. However, the manufacturer usually seals its low-carbon productions to retailers, who directly sell those productions to the customer. So, retailers are closer to consumers compared to manufacturers. When automakers, such as Toyota, Honda, etc., make fuel-efficient cars, they also need dealerships, such as 4s shops, to promote their products. This phenomenon is a very appealing situation in which manufacturers are committed to reducing emissions and producing low-carbon products. Dealerships are responsible for promoting the manufacturers' products, increasing the supply chain's low-carbon preference, and gaining more consumers' recognition. Many scholars have studied this kind of collaboration in static situations (Karray, Martín-Herrán, & Sigué, 2017).

The duration of cooperation between the manufacturer and the dealership is usually a long-term process. Such as, "almost all of the Honda's original suppliers that were selected in the late 1980s are still providing products for Honda", said Dave Nelson, a former senior vice president of American Honda Motor Co (Moody, 2012). During the long-term cooperation, customer preference may be dynamic change, which may incur dynamic changes in the manufacturer's market share. In recent years, consumers have been more susceptible to low-carbon than before.

In the traditional contract condition, the manufacturer and dealership make a deal at once without considering the variation of the low-carbon reputation of the low-carbon production. After the manufacturer and dealership sign a contract, the customer's low-carbon preference may change, which is against promoting low-carbon production according to the signed contract. Besides, in the long-term cooperation between the manufacturer and dealership, manufacturers' emission reduction and dealerships' promotional efforts need huge capital investment. In such a circumstance, the manufacturer and dealership cannot simultaneously re-balance their profits and costs, which is not beneficial for promoting low-carbon production.

Motivated by this phenomenon, this paper focuses on the study of long-term cooperation between manufacturer and dealership for exploring the design of the contract to promote selling low-carbon production considering low carbon reputation. Finding a win–win policy for collaboration between manufacturer and dealership is essential and intractable. In a dynamic market, the demands, market shares, and low-carbon reputation evolve, and the evolution depends on the decisions of the manufacturer and dealership. The dynamic change of low carbon reputation for the new energy vehicles makes the contracts between the manufacturer and dealership more complex than a static game model. The game theory methods are helpful in the decisional energy management process for the industrial sector (Barari, Agarwal, Zhang, Mahanty, & Tiwari, 2012). A differential game is more suitable than a static game model to model the problem studied in this paper. Therefore, we adopted the differential game model to model the contract between the manufacturer and the dealership for selling low-carbon production.

This paper mainly intend to solve the following questions, which are presented as follows:

- How to model contract game between the manufacturer and the dealership considering the dynamic changes of low carbon reputation of the low-carbon production?
- How to design a win–win contract the manufacturer and the dealership?
- If the win–win contract is designed, what are the conditions and optimal behaviors of the manufacturer and the dealership?

This paper's contributions are summarized as follows: (1) We studied long-term cooperation problems by designing dynamic models. We developed the state equation of low-carbon reputation influenced by the emission reduction level and the promotion of dealerships. A manufacturer's reputation for making low-carbon efforts will affect the consumer's demand for low-carbon products. The market demand will affect the profits of manufacturers and retailers. This will provide a reference for the construction of a dynamic model. (2) This paper proposed two different cost-sharing contracts, including a one-way cost-sharing (OWC) contract and a two-way cost-sharing (TWC) contract. These two types of contracts are compared and analyzed. We also examined the contract's conditions to effectively coordinate the manufacturer's actions. These studies are helpful for contract design and help enterprises to choose the contract that is suitable for them.

The remainder of the paper is organized as follows. A review of related literature is presented in Section 2, and assumptions and notations of this paper are provided in Section 3. In Section 4, both one-way and two-way cost sharing contracts are described. In Section 5, the results and discussions of the models are presented from an analytical perspective, and in Section 6, more insights are presented based on numerical experiments. Section 7 presents the extended models. Finally, conclusions and discussions of further directions in this areas are given in Section 8.

2. Literature review

We summarized the related works into five parts: consumers' low-carbon awareness, cooperation among participants in a supply chain, differential game in the supply chain, and cost-sharing contract design. The details are presented as follows.

2.1. Consumers' low-carbon awareness

With the increase in low-carbon awareness, consumers are gradually willing to buy green products at higher prices (Bai & Liu, 2013; Kotchen, 2005). Consumers would like to pay a premium on environmental-friendly wood products in the U.S (Aguilar & Vlosky, 2007). According to a survey by Nielson, about 83% of consumers care about the impacts of human activities on the environment, and for 22% of consumers, environmental-friendly products sold at a relatively higher price are acceptable (Nielsen, 2011). Echeverría, Hugo Moreira, Sepúlveda, and Wittwer (2014) found that consumers are still willing to buy milk and bread with a low carbon footprint even though these products will be 29% and 10% more expensive, respectively. Tait, Saunders, Guenther, and Rutherford (2016) noted that the more developed the economy is, the more vital willingness for low-carbon products the consumers have. This change in consumers' low-carbon awareness will inevitably affect the operation of enterprises.

The impact of consumers' low-carbon awareness on enterprises and the supply chains has been studied by many previous studies. Liu, Anderson, and Cruz (2012) focused on investigating consumers' environmental awareness and its impact on the supply chain stakeholders. The results of the study of Liu et al. (2012) showed that green supply chain stakeholders would benefit from the increased awareness. Du, Zhu, Jiao, and Ye (2015) also analyzed consumers' low-carbon preference and its influence on the supply chains with considering environmental impacts. Zhang, Wang, and You (2015) investigated consumers' environmental awareness and the corresponding impact on the order quantity, which suggested that retailers' profits increase monotonically with the increase of manufacturers' profits and that the order quantity is positively related to the rise of consumers' environmental awareness. Xia, Hao, Qin, Ji, and Yue (2018) demonstrated that manufacturers were more motivated to reduce emissions, and retailers were also likely to promote the low carbon product with consumers' increasing low-carbon awareness, which in turn enhances the profitability of the supply chain stakeholders. Through the study of the influence of consumers' environmental awareness on a retailer-led supply chain, Tong, Mu, Zhao, Mendis, and Sutherland (2019) found that consumers' environmental awareness was a crucial factor influencing manufacturers' and retailers' behaviors. Liu, et al. (2022) studied a three-level supply chain considering low-carbon consumer preferences. The studied three-level supply chain by Liu, et al. (2022) includes manufacturers, retailers, and consumers. The study of Liu, et al. (2022) found that consumers' low-carbon preferences affect the pricing of automotive supply chains. However, Liu, et al. (2022) did not consider the dynamic change in consumers' low-carbon preferences.

2.2. Cooperation among participants in a supply chain

Cooperative game theory has been successfully applied in many areas (Alamdar, Rabbani, & Heydari, 2018; Güler & Keskin, 2013; Niu, Chen, Yuan, & Xiao, 2021). This section reviews the application of cooperative game theory in supply chain management considering the carbon constraint and contract design.

Wang, Jin, Lv, and Wu (2016) studied the free-ride effect of advertising under three integrated models and designed a contract mechanism to coordinate actions involved in the process with considering a multi-channel supply chain as their research object. Wang, Zhao, and He (2016) developed two contracts to promote low-carbon cooperation between manufacturers and retailers. Karray et al. (2017) investigated the long-term effects of retail advertising on the supply chain, in which two competitive manufacturers and one retailer are involved. Similarly, Zhang and Zhang (2018) focused on the effects of cooperative advertising strategy in the context of one manufacturer and two retailers based on the multi-stage dynamic game theory. The fourth-party logistics company and the third-party logistics company usually cooperate to deliver the cargo to customers. Qi, Wang, and Xu (2018) studied a dual-channel supply chain coordination problem considering the carbon cap-and-trade regulation and developed an offline channel price discount contract for coordinating the dual-channel supply chain. Bai, Xu, and Chauhan (2020) also studied the two-stage supply chain coordination under a carbon tax policy with considering the risk aversion. Zou, Qin, and Long (2022) studied a low-carbon supply chain by extending the study of Bai et al. (2020) by considering risk aversion and carbon quota policy simultaneously. Jianmiao (2021) proposed a transfer payment contract to coordinate the emission-dependent supply. Shaban, Chan, Chung, and Qu (2021) developed a mixed wholesale option contract considering the demand imbalance between the air cargo hot-selling and underutilized routes and proposed a sequential cooperative game to formulate the mixed wholesale option contract. Wang, Huang, Feng, and Zhou (2022) studied the cooperation between the fourth-party logistics company and the third-party logistics company to design contracts to reduce tardiness. Toktaş-Palut (2022) designed Nash bargaining-based revenue-sharing contracts for coordinating the supply chains by analyzing the effects of Industry 4.0 technologies applied in supply chains. Yuan, Bi, Li, and Zhang (2022) developed an option and cost-sharing combined contract to coordinate the supply chain considering the retailer's risk aversion.

2.3. Cost-sharing contract design

As one of the widely used contracts in supply chain cooperation, cost-sharing contracts have proved to be an effective business coordination mechanism. It was introduced by Banerjee and Lin (2001) in vertical R&D cooperation, and it established the foundation for the study of cost-sharing contracts in this area. Bai, Chen, and Xu (2017) showed that a cost-sharing contract can also lead to perfect coordination as a two-part tariff contract always does. After a detailed study of the emission reduction in the context of a Make-to-Order supply chain, Xu, He, Xu, and Zhang (2017) designed a cost-sharing contract for the coordination of activities of supply chain stakeholders. By revising the demand function by considering the targeted advertising and the emissions, Liu (2019) proposed four types of cost-sharing models and studied the corresponding pricing strategies under various conditions. Very recently, Hong and Guo (2019) showed that cost-sharing contracts could improve supply chain preference in the low-carbon product supply chain. Sharma and Jain (2021) considered the fairness of cost-sharing contract design for a green supply chain. He, He, Shi, Xu, and Zhou (2020) designed a cost-sharing contract in a service supply chain to optimize the service level, emission reduction, and advertising efforts simultaneously. Designing a cost-sharing contract for a closed-loop supply chain consisting of a manufacturer and a distributor was studied by Taleizadeh, Niaki, and Alizadeh-Basban (2021) considering stochastic demand.

2.4. Differential game theory in supply chain

Chintagunta and Jain (1992) studied the dynamic of supply chain by using differential games. Their study established state equations of goodwill for manufacturers and retailers and investigated the difference in profits resulting from following coordinated and uncoordinated strategies. Zaccour (2008) investigated the different effects of the two-part wholesale price on a static marketing and a dynamic marketing channel by using differential games. Taboubi (2019) examined pricing and advertising coordination in a supply chain using differential games. Chutani and Sethi (2018) investigated the dynamic cooperative advertising between multiple manufacturers and retailers and analyzed the influence of competitive factors. Based on the differential game, Xiang and Xu (2019) developed the dynamic goodwill model for studying cooperation and coordination in a closed-loop supply chain.

The dynamic change process of product greenness was studied by Mohsin, Hossain, Tushar, Iqbal, and Hossain (2021) to explore the cooperation and coordination between a single manufacturer and a single retailer for establishing a green supply chain. Cheng and Ding (2021) studied competitive supply chains and corporate social responsibilities in supply chains considering the dynamic change of corporate social responsibility.

2.5. Research gaps

In this subsection, we summarize the research gaps of this paper and previous studies. The research gaps are divided into four parts which show as follow.

Research gaps of consumers' low-carbon awareness: The literature review has shown that previous studies mainly focused on the problems in a single period. The dynamic changes in customers' low-carbon preference is an essential issue, which has been emphasized by recent studies (Wei & Wang, 2021). From previous studies, we found consumers' low-carbon preference influences the market share of low-carbon products, and consumers' low-carbon preferences will increase demand. Hence, it is essential to investigate the effect of dynamic change on consumers' low-carbon preferences in promoting low-carbon production.

Research gaps of cooperation among participants in a supply chain: Cooperation among participants through contracts in the low-carbon has been studied by many previous studies. In this paper, the participants in the supply chain are manufacturers and dealerships, considering the customer's low-carbon preferences. Usually, the manufacturers are leaders, and the dealerships are followers of the Stackelberg game. The manufacturers have a dominant position in the market, and many previous studies the one-way sharing between the leaders and followers. This paper develops a two-way cost-sharing between manufacturers and dealerships.

Research gaps of cost-sharing contract design: From the above previous study analysis, we also found that a cost-sharing contract is an effective coordination mechanism for contract design in supply chain management. The cost-sharing contracts discussed above focused on the one-way cost-sharing contract. When designing a cost-sharing contract, which at least involves two participants, the previous studies minimized or maximized one participant's cost or profit. The goal of the two participants is usually a trade-off. Hence, balancing the goal of the two participants is also an essential issue when designing the cost-sharing contract. The cost-sharing contract ensures a Pareto improvement of channel profits and improves the manufacturer's environmental protection efforts (Yang & Gong, 2021). In contrast, limited studies on the two-way cost-sharing contract could find a better Pareto improvement solution. In this paper, we consider both one-way sharing and two-way sharing contracts and compare the two strategies.

Research gaps of differential game theory in supply chain: The previous studies considered the dynamics of the games, such as the Stackelberg game, without considering the continuity. To address this issue, in this paper, we study the dynamics of the game with considering continuity. The manufacturer and dealership interaction is the Stackelberg game that also assesses the dynamics of the interaction. Such models are established by differential equations, considering the customer's low-carbon preference dynamics.

3. Problem assumptions and notations

3.1. Description of the problem

With the environmental consciousness increase of customers, customers' attitude toward low-carbon products becomes more and more positive. The manufacturers have been focused on developing new technology to design advanced low-carbon products for sales promotion and reducing carbon emissions simultaneously. The business model for selling low-carbon products can be classified into a traditional dealership and direct distribution models. In the conventional dealership model, the traditional dealership is a dealership, such as, 4S stores for sale, sparepart, service, and survey. With the development of online sales platforms, the manufacturers also started to adopt direct distribution to sell low-carbon products through online platforms (Biller, Chan, Simchi-Levi, & Swann, 2005; Bohnsack, Pinkse, & Kolk, 2014; Martins, Rindova, & Greenbaum, 2015). In the direct distribution model, the dealership could be the manufacturer itself. Hence, the manufacturer has leadership in the automation market compared with the dealership. The manufacturer's emission reduction effort and the dealership's low-carbon product promotion effort can win consumers with environmental awareness. Here, we call this recognition a low-carbon reputation. The low-carbon reputation increases with the effort made by the manufacturer and the dealership.

To encourage the dealership to promote low-carbon products, the manufacturer provides financial support to the dealership, which is called a one-way cost-sharing contract. This paper also considers another type of contract called a two-way cost-sharing contract. In the TWC contract, the manufacturer shares some portion of the promotion cost of the low-carbon product from the dealership. In turn, the dealership shares some emission reduction costs with the manufacturer. By modeling this problem and solving the model, we obtain the contract conditions that the manufacturer and dealership can use for contract design. The problems are shown in 2.

Similar to the concession contract design problem between port authority and container port (Zhou & Kim, 2020, 2021), the manufacturer has a dominant position in the auto market compared with the dealership. Hence, this paper assumes that the manufacturer is a leader and the dealership acts as a follower. The manufacturer chooses a contract type and gives the contract's parameters first. Then the dealership makes optimal responses based on the manufacturer's given contract. It is reasonable to assume Assumption 1, which is presented as follows:

Assumption 1. This paper considers a manufacturer and a dealership. The relationship between the manufacturer and the dealership is modeled as a Stackelberg game.

3.2. Relationship between reputation, demand and cost

Before introducing the relationship between reputation, demand and cost, the notations, and decision variables used in this paper are listed in Table 1.

The low-carbon awareness of consumers is usually affected by many factors and Penz, Hartl, and Hofmann (2019) investigated consumer choice of low carbon footprint goods. BrandZ did a survey and concluded that environmental reputation directly influences about 2% of all sales.[2] In this study, we assume that the customer demand will be indirectly affected by emissions reduction effort of the manufacturer and the low carbon promotion effort of the dealership. Komarek, Lupi, Kaplowitz, and Thorp (2013) adopted a linear regression to analyze the energy management plan's impact on green reputation. In this paper, a linear demand model is adopted. The primary demand for products without emissions reductions and low-carbon promotion and regardless of low-carbon reputation is a. The demand is positively related to the $G(t)$ with a constant θ.

Assumption 2. The demand function of the low-carbon product for the dealership is linear related to the low-carbon reputation, which is defined as follow:

$$Q(t) = a + \theta G(t) \tag{1}$$

The reputation of the low-carbon product is essential for selling, and the reputation is always dynamically changing with the outer and inner environmental adjustments (Bolton, Greiner, & Ockenfels, 2013). The Reputation.com ranks the top auto brands and dealerships to help manufacturers analyze their selling. When buying a new energy vehicle, consumers will be affected by the ranks of the leading auto brands.

[2] BrandZ is the largest global brand equity platform, and the details can be found by accessing https://www.theguardian.com/environment/2011/jul/01/carbon-trust-research-footprint-consumer-demand.

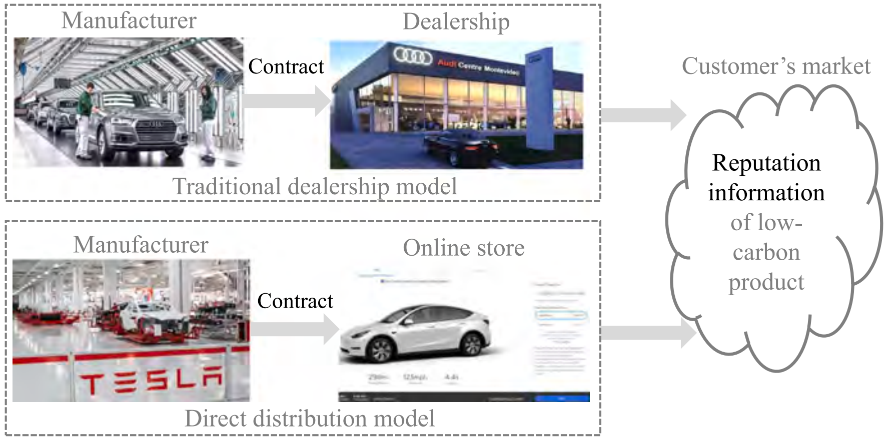


Fig. 2. The problem description.

Table 1
Notations and decision variables.

Notations	
t	Time period.
$G(t)$	Low carbon reputation of the supply chain at time t.
$G(0)$	Initial value of the low carbon reputation.
$Q(t)$	Demand function.
π_M	Marginal profit obtained by the manufacturer.
π_R	Marginal profit obtained by the dealership.
η_M	Manufacturer's cost coefficient related to emission reduction, $\eta_M > 0$.
η_R	Dealership's cost coefficient related to the promotion of low-carbon product, $\eta_R > 0$.
γ_M	Influence coefficient of manufacturer's emission reduction efforts on low-carbon reputation.
γ_R	Influence coefficient of dealership's low-carbon promotion efforts on low-carbon reputation.
δ	Attenuation coefficient of low-carbon reputation, $\delta > 0$.
a	Basic market demand disregarding emission reduction and promotion, $a > 0$.
θ	Low carbon reputation sensitivity coefficient of consumers, $\theta > 0$.
ρ	Discount rate of profit, $\rho > 0$.
Decision variables	
$E_M(t)$	Manufacturer's effort on the emission reduction at time t, $E_M(t) \geq 0$.
$E_R(t)$	Dealership's effort on the promotion of low-carbon products at time t, $E_R(t) \geq 0$.
$X(t)$	Cost proportion shared by the manufacturer from the dealership, $0 \leq X(t) \leq 1$.
$Y(t)$	Cost proportion shared by the dealership from the manufacturer $0 \leq (t) \leq 1$.

Assumption 3. The reputation of the low-carbon product is determined by the manufacturer's emissions reduction effort and the dealership's low-carbon promotion effort simultaneously. This dynamic process can be described by a differential equation which is presented as follows:

$$\dot{G}(t) = \gamma_M E_M(t) + \gamma_R E_R(t) - \delta G(t), \tag{2}$$

where $G(t)$ is the low-carbon reputation at time t and the low carbon reputation in the beginning of taking actions is $G(0) = G_0 \geq 0$. If the manufacturer does not reduce emissions and the dealership does not carry out promotion, the low-carbon reputation will be lowered by $\delta G(t)$. If the manufacturer's effort is $E_M(t)$, the reputation will increase by $\gamma_M E_M(t)$. If the dealership's effort is $E_R(t)$, the reputation will increase by $\gamma_R E_R(t)$. This differential equation (2) has been used in many previous studies (Jørgensen, Taboubi, & Zaccour, 2003a; Taboubi, 2019).

Assumption 4. The extra costs of the manufacturer and the dealership to take actions to reduce emissions are assumed to be quadratic functions. The traditional marginal cost is zero.

Quadratic extra cost functions are popular and have been adopted by many previous studies (Ghosh & Shah, 2012; Wang, Zhang, Fan, & Zhu, 2020). In this paper, the extra cost for the manufacturer's efforts is defined as follows:

$$C_M\left(E_M(t)\right) = \frac{\eta_M\left(E_M(t)\right)^2}{2}. \tag{3}$$

The extra cost for the efforts made by the dealership is

$$C_R\left(E_R(t)\right) = \frac{\eta_R\left(E_R(t)\right)^2}{2}, \tag{4}$$

which is considered to be one-time investments.

4. The contract models

The contract can be classified into four categories based on the business models and cost-sharing rules, which shows in Table 2.

This section discusses the models and derives the equilibrium for the manufacturer and the dealership considering the case A, B, C, and D, shown in Table 2. This section also analyzes and discusses the four cases and the optimal decisions, such as the sensitivity of parameters and decision variables.

4.1. Centralized decision case (Case A)

The direct distribution model is considered a centralized decision case in which the manufacturer and the dealership simultaneously coordinate their decisions to maximize profits. In practice, it is challenging to coordinate the manufacturer and the dealership because the dealership usually does not belongs to the manufacturer. The centralized decision case is an ideal situation and can be used as a benchmark for contract analysis, which has been used in the literature (Ghosh & Shah, 2015; Modak & Kelle, 2019).

Table 2
Four contract cases.

Contract case	Business model	Decision model	Cost-sharing
Case A	Direct distribution model	Centralize	N/A
Case B	Traditional dealership model	Decentralize	N/A
Case C	Traditional dealership model	Decentralize	One-way sharing
Case D	Traditional dealership model	Decentralize	Two-way sharing

In the centralized decision case, the total profit of the manufacturer and the dealership can be formulated as follows:

$$J_{sc}^{sA} = \max_{E_M^{sA}, E_R^{sA}} \int_0^{\infty} e^{-\rho t}\{(\pi_M + \pi_R)Q - C_M(E_M^{sA}) - C_R(E_R^{sA})\}dt. \quad (5)$$

E_M^{sA} and E_R^{sA} represent the profit of the manufacturer and the dealership, respectively. J_{sc}^{sA} denotes the total profit obtained by the manufacturer and the dealership under the coordination decision making (The superscript A represents case A for centralized decision case and the superscript s represents single-channel).

Eq. (5) is Hamilton–Jacobi–Bellman (HJB) equation. Similar to the study of Jørgensen et al. (2003a), in this paper, the parameters in Eq. (5) are independent of time. For the convenience of writing, the time t is omitted in the following sections.

Lemma 1. *Under the centralized decision model, the equilibrium, optimal trajectory of low carbon reputation, the optimal profit of the manufacturer and the dealership are presented as follows.*

(i) The equilibrium strategies of the manufacturer and the dealership are formulated in Eqs. (6) and (7)*, respectively.*

$$E_M^{sA*} = \frac{\gamma_M(\pi_M + \pi_R)\theta}{\eta_M(\rho + \delta)} \quad (6)$$

$$E_R^{sA*} = \frac{\gamma_R(\pi_M + \pi_R)\theta}{\eta_R(\rho + \delta)} \quad (7)$$

(ii) The optimal trajectory of low carbon reputation is formulated in Eq. (8).

$$G^{sA*}(t) = (G_0 + \frac{B^{sA}}{A^{sA}})e^{A^{sA}t} - \frac{B^{sA}}{A^{sA}}, \quad (8)$$

where $A^{sA} = -\delta$, $B^{sA} = \gamma_M E_M^{sA*} + \gamma_R E_R^{sA*}$.

(iii) The optimal total profit of the manufacturer and the dealership is formulated in Eq. (9).

$$J_{sc}^{sA*} = e^{-\rho t}(a_3^{sA*} G^{sA*}(t) + b_3^{sA*}), \quad (9)$$

where $a_3^{sA*} = \frac{(\pi_M+\pi_R)\theta}{\rho+\delta}$, $b_3^{sA*} = \frac{\gamma_M^2(\pi_M+\pi_R)^2\theta^2}{2\eta_M\rho(\rho+\delta)} + \frac{\gamma_R^2(\pi_M+\pi_R)^2\theta^2}{2\eta_R\rho(\rho+\delta)} + \frac{(\pi_M+\pi_R)a}{\rho}$

The proof of Lemma 1 is shown in Appendix A.

Lemma 1 gives the optimal profit function for the manufacturer and the dealership. In the following, we explore how the parameters, including η_M, η_R, γ_M, γ_R and θ, affect the profit functions given in Lemma 1.

Proposition 1. *(i)* $\frac{\partial E_M^{sA*}}{\partial \eta_M} < 0$, $\frac{\partial E_R^{sA*}}{\partial \eta_R} < 0$; *(ii)* $\frac{\partial E_M^{sA*}}{\partial \gamma_M} > 0$, $\frac{\partial E_R^{sA*}}{\partial \gamma_R} > 0$; *(iii)* $\frac{\partial E_M^{sA*}}{\partial \theta} > 0$; $\frac{\partial E_R^{sA*}}{\partial \theta} > 0$; *(iv)* $\frac{\partial E_M^{sA^*}}{\partial \delta} \leq 0$ *and* $\frac{\partial E_R^{sA^*}}{\partial \delta} \leq 0$;

The Proposition 1 (i) states that the manufacturer's emission reduction efforts decrease with η_M increasing and the dealership's low-carbon promotion efforts decrease with η_R increasing. Proposition 1 (ii) shows that the manufacturer's emissions reduction efforts (or the dealership's low-carbon promotion efforts) are also affected by the coefficient of γ_M(or γ_R). The manufacturer (or the dealership) is more willing to work hard as γ_M(or γ_R) increases. Proposition 1(iii) shows that the manufacturer's emissions reduction efforts and the dealership's low-carbon promotion efforts increase as θ increases.

When the emission reduction difficulty is more serious, the enterprise is less willing to make emission reduction efforts. On the contrary, the lower the emission reduction difficulty, the enterprise is more inclined to make more emission reduction efforts. The government could formulate the corresponding policies to encourage enterprises to reduce emissions according to different circumstances. Results revealed by Proposition 1(ii) and Proposition 1(iii) are both related to consumers' low-carbon awareness. Consumers' strong awareness of low-carbon products will push the manufacturer and the retailers to enhance their low-carbon reputation. With a better reputation, demand for the products will become more significant. Therefore, improving consumers' low-carbon awareness is more conducive to promoting the development of a low-carbon economy. This is the main reason why governments have been active in building strong low-carbon awareness among consumers. From the following equations, we can easily find that iv always holds. The faster the decay of a low-carbon reputation, the less conducive it is for companies to reduce emissions. Society should establish an excellent low-carbon reputation mechanism to enable enterprises to maintain continuous low-carbon reputation motivation.

$$\frac{\partial E_R^{sA^*}}{\partial \delta} = -\frac{(\pi_M + \pi_R)\gamma_R\theta}{\eta_R(\rho+\delta)^2} \quad (10)$$

$$\frac{\partial E_M^{sA^*}}{\partial \delta} = -\frac{(\pi_M + \pi_R)\gamma_M\theta}{\eta_M(\rho+\delta)^2} \quad (11)$$

$$\frac{\partial E_M^{sB^*}}{\partial \delta} = -\frac{\pi_M\gamma_M\theta}{\eta_M(\rho+\delta)^2} \quad (12)$$

$$\frac{\partial E_R^{sB^*}}{\partial \delta} = -\frac{\pi_R\gamma_R\theta}{\eta_R(\rho+\delta)^2} \quad (13)$$

4.2. Decentralized decision case without cost sharing (Case B)

This section analyzes the decentralized decision case without cost-sharing. In this case, the manufacturer and the dealership make decisions independently. First, the manufacturer determines its emission reduction efforts. Then the dealership decides on low-carbon promotion efforts based on the manufacturer's decision. The manufacturer and the dealership intend to optimize the present value of their profits. (Superscript N represents the decentralized decision case without a cost-sharing contract)

The objective functions of the manufacturer and the dealership are formulated as follows:

$$J_M^{sB} = \max_{E_M^{sB}(t)} \int_0^{\infty} e^{-\rho t}\{\pi_M Q - C_M(E_M^{sB}(t))\}dt \quad (14)$$

$$J_R^{sB} = \max_{E_R^{sB}(t)} \int_0^{\infty} e^{-\rho t}\{\pi_R Q - C_R(E_R^{sB}(t))\}dt \quad (15)$$

By solving the above two objective functions simultaneously, we can obtain the optimal solutions for the manufacturer and the dealership, which summarized in Lemma 2.

Lemma 2. *(i) The equilibrium strategies for the manufacturer and the dealership are* $(E_M^{sB*}, {E_R}^{sB*})$*, respectively.*

$$E_M^{sB*} = \frac{\gamma_M\pi_M\theta}{\eta_M(\delta+\rho)}. \quad (16)$$

$$E_R^{sB*} = \frac{\gamma_R\pi_R\theta}{\eta_R(\delta+\rho)}. \quad (17)$$

(ii) The optimal trajectory of low-carbon reputation is:

$$G^{sB*}(t) = (G_0 + \frac{B^{sB}}{A^{sB}})e^{A^{sB}t} - \frac{B^{sB}}{A^{sB}}, \quad (18)$$

where $A^{sB} = -\delta$ *and* $B^{sB} = \gamma_M E_M^{sB*} + \gamma_R E_R^{sB*}$.

(iii) The optimal profits for the manufacturer and the dealership are

$$J_M^{sB*} = e^{-\rho t}(a_1^{sB*} G + b_1^{sB*}) \tag{19}$$

and

$$J_R^{sB*} = e^{-\rho t}(a_2^{sB*} G + b_2^{sB*}), \tag{20}$$

where $a_1^{sB*}, b_1^{sB*}, a_2^{sB*}$, *and* b_2^{sB*} *are shown in Eq.* (21).

$$\begin{cases} a_1^{sB*} = \frac{\pi_M \theta}{\delta+\rho} \quad and \quad b_1^{sB*} = \frac{\pi_M a}{\rho} + \frac{\pi_M{}^2 \theta^2 r_M{}^2}{2\eta_M \rho(\delta+\rho)^2} + \frac{\pi_M \pi_R \theta^2 r_R{}^2}{\eta_R \rho(\delta+\rho)^2} \\ a_2^{sB*} = \frac{\pi_R \theta}{\delta+\rho} \quad and \quad b_2^{sB*} = \frac{\pi_R a}{\rho} + \frac{\pi_R{}^2 \theta^2 r_R{}^2}{2\eta_R \rho(\delta+\rho)^2} + \frac{\pi_R \pi_M \theta^2 r_M{}^2}{\eta_M \rho(\delta+\rho)^2} \end{cases} \tag{21}$$

The proof of Lemma 2 is shown in Appendix B.

By using the solutions obtained by Lemma 2, we analyze the equilibrium strategies for the manufacturer and dealership considering the variation of the input parameters including η_R, γ_M and θ. Through analyzing, we conclude Proposition 2, which is presented as follows:

Proposition 2. *(i)* $\frac{\partial E_M^{sB*}}{\partial \eta_M} < 0$, $\frac{\partial E_R^{sB*}}{\partial \eta_R} < 0$; *(ii)* $\frac{\partial E_M^{sB*}}{\partial \gamma_M} > 0$, $\frac{\partial E_R^{sB*}}{\partial \gamma_R} > 0$, *(iii)* $\frac{\partial E_M^{sB*}}{\partial \theta} > 0$, $\frac{\partial E_R^{sB*}}{\partial \theta} > 0$.

With Proposition 2, we know that the manufacturer's emissions reduction efforts and the dealership's low-carbon promotion efforts are sensitive to η_R, γ_M and θ, which are the same as the situation of centralized decision model (Case A).

Corollary 1. *Comparing the optimal strategies in the above two cases (Case A and Case B), we can find that,* $E_M^{sA*} > E_M^{sB*}, E_R^{sA*} > E_R^{sB*}$

Corollary 1 indicates the manufacturer's emission reduction efforts and the dealership's low-carbon promotion efforts are lower than the situation with the centralized decision.

In Case B, the contract is a one-shot deal. However, consumers' low-carbon awareness usually changes with the dynamic variation of the market environment. To reduce the risk of dynamic change in consumers' low-carbon awareness, the manufacturer and the dealership could better cooperate to share some costs. Considering that the manufacturer is a leader who takes the initiative to promote the dealership's efforts, we propose an OWC contract where the manufacturer shares the promotion costs of the dealership. We discuss the OWC in the following subsection.

4.3. One-way cost sharing contract (Case C)

In this case C, the manufacturer is willing to share some of the low-carbon promotion cost for the dealership to create an incentive. So the manufacturer determines E_M^{sC} and X^{sC} simultaneously, then the dealership decides E_R^{sC} by using the given parameters including $E_{M^{sC}}$ and X^{sC}. (Superscript C represents case C.)

Under case C, the objective functions of the manufacturer and the dealership are formulated as follows:

$$J_M^{sC} = \max_{E_M^{sC}, X^{sC}} \int_0^\infty e^{-\rho t}\{\pi_M Q - C_M(E_M^{sC}) - X^{sC} C_R(E_R^{sC})\} dt. \tag{22}$$

$$J_R^{sC} = \max_{E_R^{sC}} \int_0^\infty e^{-\rho t}\{\pi_R Q - (1 - X^{sC}) C_R(E_R^{sC})\} dt. \tag{23}$$

By solving the above to objective functions, we can conclude Lemma 3, which shows as follows.

Lemma 3. *(i) The optimal trajectory of low-carbon reputation is:*

$$G^{sC*}(t) = (G_0 + \frac{B^{sC}}{A^{sC}}) e^{At} - \frac{B^{sC}}{A^{sC}}. \tag{24}$$

(ii) The equilibrium strategies of the manufacturer and the dealership are $((E_M^{sC*}, X^{sC*}), E_R^{sC*})$.

$$E_M^{sC*} = \frac{\gamma_M \pi_M \theta}{\eta_M(\rho + \delta)} \tag{25}$$

$$X^{sC*} = \frac{2\pi_M - \pi_R}{2\pi_M + \pi_R} \tag{26}$$

$$E_R^{sC*} = \frac{\gamma_R(2\pi_M \theta + \pi_R \theta)}{2\eta_R(\rho + \delta)} \tag{27}$$

(iii) The optimal profits of the manufacturer and the dealership are as follows:

$$J_M^{sC*} = e^{-\rho t} a_1^{sC*} G + b_1^{sC*}. \tag{28}$$

$$J_R^{sC*} = e^{-\rho t} a_2^{sC*} G + b_2^{sC*}, \tag{29}$$

where $A^{sC} = -\delta$ *and* $B^{sC} = \gamma_M E_M^{sC*} + \gamma_R E_R^{sC*}$.

$$\begin{cases} a_1^{S*} = \frac{\pi_M \theta}{\rho+\delta} \quad and \quad b_1^{sC*} = \frac{[\gamma_M a_1^{sC*}]^2}{2\eta_M \rho} + \frac{\pi_M a}{\rho} + \frac{\gamma_R{}^2 [2a_1^{sC*} + a_2^{S*}]^2}{8\eta_R \rho} \\ a_2^{sC*} = \frac{\pi_R \theta}{\rho+\delta} \quad and \quad b_2^{sC*} = \frac{\gamma_M^2 a_2^{S*} a_1^{sC*}}{\eta_M \rho} + \frac{\pi_R a}{\rho} + \frac{\gamma_R^2 a_2^{S*} [2a_1^{S*} + a_2^{S*}]}{4\eta_R \rho} \end{cases} \tag{30}$$

The proof of Lemma 3 is shown in Appendix C

Corollary 2. *Comparing the optimal strategies in the above three cases, it can be found that the following two equations always hold.*

$$E_M^{sB*} = E_M^{sC*} < E_M^{sA*} \tag{31}$$

$$E_R^{sB*} < E_R^{sC*} < E_R^{sA*} \tag{32}$$

From Corollary 2, we find several interesting conclusions: (1) Compared with case B, E_M^{sC*} has not changed in case C. (2) E_R^{sC*} has been enhanced compared with case B. (3) Both E_M^{sC*} and E_R^{sC*} failed to reach the situation in centralized decision after using the OWC contract. Although the supply chain achieved Pareto improvement, it is still not being coordinated. According to Corollaries 1 and 2, we find that E_M^{sC*} could be more enhanced when the manufacturer requires the dealership to share its emissions reduction cost in their contract. To achieve coordination between both sides of the manufacturer and the dealership, we introduce two-way cost-sharing in the next subsection.

4.4. Two-way cost sharing contract (Case D)

Realizing the problem found in Corollary 2, we design a new contract option that pursues TWC. Under case D, the manufacturer shares the dealership's low-carbon promotion cost and requires the dealership to share its emissions reduction cost. The manufacturer needs to determine the contract parameters (X, Y), where X represents the amount that the manufacturer shares in the dealership's low carbon promotion cost and Y are the amount that the manufacturer requires the dealership to share for its emissions reduction effort. Then the manufacturer and the dealership make their own decisions based on the contract parameters (superscript D represents case D).

Under the case D, the objective functions of the manufacturer and the dealership are defined as follows:

$$J_M^{sD} = \max_{E_M^{sD}, X^{sD}, Y^{sD}} \int_0^\infty e^{-\rho t}\{\pi_M Q - (1 - Y^{sD}) C_M(E_M^{sD}) - X^{sD} C_R(E_R^{sD})\} dt \tag{33}$$

$$J_R^{sD} = \max_{E_R^{sD}} \int_0^\infty e^{-\rho t}\{\pi_R Q - Y^{sD} C_M(E_M^{sD}) - (1 - X^{sD}) C(E_R^{sD})\} dt \tag{34}$$

After solving the above two objective functions, the equilibrium solutions can be obtained which shows in Lemma 4.

Table 3
Summary of the equilibrium strategies and its corresponding profits for case A, B, C and D.

Case	Equilibrium strategies		Profit	
	Manufacturer	Dealership	Manufacturer	Dealership
A	$\frac{\gamma_M(\pi_M+\pi_R)\theta}{\eta_M(\rho+\delta)}$	$\frac{\gamma_R(\pi_M+\pi_R)\theta}{\eta_R(\rho+\delta)}$	$e^{-\rho t}(a_3^{sA*}G^{sA*}(t)+b_3^{sA*})$	
B	$\frac{\gamma_M\pi_M\theta}{\eta_M(\delta+\rho)}$	$\frac{\gamma_R\pi_R\theta}{\eta_R(\delta+\rho)}$	$e^{-\rho t}(a_1^{sB*}G+b_1^{sB*})$	$e^{-\rho t}(a_2^{sB*}G+b_2^{sB*})$
C	$\frac{\gamma_M\pi_M\theta}{\eta_M(\rho+\delta)}$	$\frac{\gamma_R(2\pi_M\theta+\pi_R\theta)}{2\eta_R(\rho+\delta)}$	$e^{-\rho t}a_1^{sC*}G+b_1^{sC*}$	$e^{-\rho t}a_2^{sC*}G+b_2^{sC*}$
D	$\frac{\gamma_M\pi_M\theta}{\eta_M(1-Y^{sD*})(\rho+\delta)}$	$\frac{\gamma_R\pi_R\theta}{\eta_R(1-X^{sD*})(\rho+\delta)}$	$e^{-\rho t}a_1^{sD*}G+b_1^{sD*}$	$e^{-\rho t}a_2^{sD*}G+b_2^{sD*}$

Lemma 4. *(i) The optimal trajectory of low carbon reputation shows in Eq. (35).*

$$G^{sD*}(t)=(G_0+\frac{B^{sD}}{A^{sD}})e^{At}-\frac{B^{sD}}{A^{sD}}. \tag{35}$$

(ii) The equilibrium strategies of the manufacturer and the dealership are formulated in Eqs. (36) and (37)*, respectively.*

$$E_M^{sD*}=\frac{\gamma_M\pi_M\theta}{\eta_M(1-Y^{sD*})(\rho+\delta)}. \tag{36}$$

$$E_R^{sD*}=\frac{\gamma_R\pi_R\theta}{\eta_R(1-X^{sD*})(\rho+\delta)}. \tag{37}$$

(iii) The optimal profits of the manufacturer and the dealership are formulated in Eqs. (38) and (39)*, respectively.*

$$J_M^{sD*}=e^{-\rho t}a_1^{sD*}G+b_1^{sD*}. \tag{38}$$

$$J_R^{sD*}=e^{-\rho t}a_2^{sD*}G+b_2^{sD*}, \tag{39}$$

where $A^{sD}=-\delta$ *and* $B^{sD}=\gamma_M E_M^{sD*}+\gamma_R E_R^{sD*}$.

$$\begin{cases} a_1^{sD*}=\frac{\pi_M\theta}{\rho+\delta} \quad \text{and} \quad b_1^{sD*}=\frac{\pi_M a}{\rho}-\frac{X^{sD}\gamma_R^2\left(a_2^{sD}\right)^2}{2\eta_R(1-X^{sD})^2\rho}+\frac{\gamma_M^2\left(a_1^{sD}\right)^2}{2\eta_M(1-Y^{sD})\rho}+\frac{\gamma_R{}^2a_1^{sD}a_2^{sD}}{\eta_R(1-X^{sD})\rho} \\ a_2^{sD*}=\frac{\pi_R\theta}{\rho+\delta} \quad \text{and} \quad b_2^{sD*}=\frac{\pi_R a}{\rho}-\frac{Y^{sD}\gamma_M^2\left(a_1^{sD}\right)^2}{2\eta_M(1-Y^{sD})^2\rho}+\frac{\gamma_R^2\left(a_2^{sD}\right)^2}{2\eta_R(1-X^{sD})\rho}+\frac{\gamma_M^2a_1^{sD}a_2^{sD}}{\eta_M\left(1-Y^{sD}\right)\rho} \end{cases} \tag{40}$$

The proof of Lemma 4 is shown in Appendix D.

According to Lemmas 4 and 1, we can have Proposition 3.

Proposition 3. *Under the TWC contract, the contract parameters are*

$$X^{sD*}=\frac{\pi_M}{\pi_M+\pi_R}, Y^{sD*}=\frac{\pi_R}{\pi_M+\pi_R} \tag{41}$$

From Lemmas 1 and 4, we obtain the E_M^{sD*}, E_R^{sD*}, E_M^{sA*}, and E_R^{sA*}. Let $E_M^{sD*}=E_M^{sA*}$, $E_R^{sD*}=E_R^{sA*}$ and solve the equations. We can finally get $X^{sD*}=\frac{\pi_M}{\pi_M+\pi_R}$, $Y^{sD*}=\frac{\pi_R}{\pi_M+\pi_R}$. Proof of Proposition 3 is very intuitive and we omit the details of the proof process.

From Proposition 3, we know that if the manufacture and the dealership could mutually share some of the cost, they are incentive to coordinate. The level of their efforts will be the same as that in the centralized decision model. The TWC contract eliminates the double marginalization effect and realizes supply chain coordination. When establishing a contract between enterprises, the incentive effect of the contract on both parties should be considered, so it is easier to achieve a win–win goal.

5. Model analysis

From Section 4, we know the equilibrium for each contract case. We summarized the equilibrium strategies and profit of the manufacturer and the dealership in Table 3, which is presented as follow.

In this section, we analyze the conditions for each contract case that preferred by the manufacture and dealership.

5.1. Analysis of the manufacture's emissions reduction efforts

First, we discuss variation of the manufacture's efforts for reducing emissions, which can give some insight of the impact on contract options.

Proposition 4. $E_M^{sB*}=E_M^{sC*}<E_M^{sA*}=E_M^{sD*}$

From the above Proposition 4, we can find that the manufacturer's emission reduction efforts are the same under the decentralized decision case without cost-sharing and the OWC. The manufacturer does not share any promotion cost for the low-carbon product in cases B and C. Hence, the manufacturer's emission reduction efforts with OWC are lower than the manufacturer's emission reduction efforts obtained by the TWC. We can also find that the manufacturer's emission reduction efforts under TWC are equal to those obtained under the centralized decision model. The two-way cost-sharing contract can motivate both parties to make more efforts. There is no incentive for the party who is not sharing the cost in a one-way cost-sharing contract. When making a contract, the manufacturer should consider the motivational effect. Two-way cost-sharing can motivate both parties to make more efforts. One-way cost-sharing cannot encourage the party who has not shared the cost. When formulating a contract, the leading party of the contract should also take into account its incentives.

5.2. Analysis of the dealership's efforts in promoting low-carbon products

Next, we discuss changes in the dealership's efforts in promoting low-carbon product, which help for analyzing the contract options.

Proposition 5. *For arbitrary* π_M *and* π_R, $E_R^{sA*}=E_R^{sD*}$ *always hold. If* $2\pi_M>\pi_R$*, then* $E_R^{sB*}<E_R^{sC*}<E_R^{sA*}$ *(or* E_R^{sD*}*), otherwise,* $E_R^{sC*}<E_R^{sB*}<E_R^{sA*}$ *(or* E_R^{sD*}*).*

Proof of Proposition 5 is shown in Appendix E.

Proposition 5 shows that the dealership's promotion efforts have different characteristics with the varying of π_M and π_R. From Proposition 5, we conclude that the π_M and π_R does not affect the identical equation of E_R^{sA*} and E_R^{sD*}. In a TWC contract, the dealership always makes the best efforts that equal the centralized scenario level. However, with an OWC, the dealership can have the best efforts or perform better than that without a cost-sharing contract, which depends on the manufacturer and the dealership's respective marginal profits. Hence, the manufacturer cannot always expect to enhance the dealership's promotion efforts with the OWC contract. When a one-way cost-sharing contract is signed, if the manufacturer's profit margin is small compared with the retailer, the manufacturer will require the retailer to share its costs, resulting in less enthusiasm for the retailer's low-carbon promotion efforts. The upstream sharing of downstream costs only applies to enterprises with larger upstream profit margins.

Under the TWC contract, $X^{sD*}=\pi_M/(\pi_M+\pi_R)>0, Y^{sD*}=\pi_R/(\pi_M+\pi_R)>0$, the cost of the manufacturer and the dealership will both decline, which leads to a result that both of them are willing to make greater efforts. When the manufacturer wants to enter into an OWC contract, it needs to pay attention to comparing its profit with that of the dealership. It is accessible for the manufacturer to enter into the OWC contract if he has a high-profit margin; otherwise, it is not accessible.

5.3. Conditions and options for selecting contracts

In this section, we solve two questions: (1) Do the manufacturer and dealership prefer a cost-sharing contract compared with the decentralized decision case without cost-sharing? (2) If the manufacturer and dealership prefer a cost-sharing contract, one-way sharing, or two-way sharing, will they choose?

First, we analyze the condition that the manufacturer and dealership choose the OWC contract. When the manufacturer and dealership choose the OWC contract, the manufacturer's profit and the dealership's profit under the OWC contract should be greater than the profit obtained without a cost-sharing contract simultaneously.

When the contract is established, all the participation's profit under the OWC contract cannot be lower than that without a cost-sharing contract, which means that the following in-equation must be held.

$$V_M^{sC*}(G) - V_M^{sB*}(G) > 0 \ \ and \ \ V_R^{S*}(G) - V_R^{sB*}(G) > 0. \tag{42}$$

From last section, we can easily obtain that

$$V_M^{sC*}(G) - V_M^{sB*}(G) = \frac{\theta^2\gamma_R^2\left(2\pi_M - \pi_R\right)}{8\eta_r\rho(\rho+\delta)^2} \tag{43}$$

$$V_R^{sC*}(G) - V_R^{sB*}(G) = \frac{\theta^2\gamma_R^2\pi_R\left(2\pi_M - \pi_R\right)}{4\eta_r\rho(\rho+\delta)^2} \tag{44}$$

Then, we can concluded that if $2\pi_M > \pi_R$, $V_M^{sC*}(G) - V_M^{sB*}(G) > 0$ and $V_R^{sC*}(G) - V_R^{sB*}(G) > 0$ are hold. Finally, the OWC contract is established. By analyzing the profit of the manufacturer under OWC and cost-sharing contract, we conclude Proposition 6, which shows as follow.

Proposition 6. *When $2\pi_M > \pi_R$, the manufacturer and dealership will accept the OWC contract.*

From Proposition 6, when the manufacturer's marginal profit is high enough relative to the marginal profit of the dealership, it is profitable for the manufacturer to choose the OWC contract. Under the OWC contract, the manufacturer and the dealership will improve their profits simultaneously. Thus, the dealership will prefer the OWC contract that is suggested by the manufacturer. The one-way cost-sharing contract is suitable for cooperation between enterprises with larger upstream profit margins.

Next, the conditions of a TWC contract are analyzed, which is shown in Proposition 7. When the TWC contract is established, the participation constraint of the contract must be satisfied, which is $V_M^{sD*}(G) - V_M^{sB*}(G) > 0$ and $V_R^{sD*}(G) - V_R^{sB*}(G) > 0$. Under a TWC contract, the manufacturer and the dealership are simultaneously more profitable than without a cost-sharing contract. Although two-way cost-sharing can achieve better coordination effect, the requirements are more stringent. It is easier to implement one-way cost-sharing contract compared with the two-way cost-sharing contract.

Let $f_{Mdn} = V_M^{sD*}(G) - V_M^{sB*}(G)$, we can derive the expression of f_{Mdn}, which shows as follow:

$$f_{Mdn} = \frac{\theta^2\pi_M\left(\eta_M\pi_M\gamma_R^2 - \eta_M\pi_R\gamma_R^2 + \eta_R\pi_R\gamma_M^2 + 2\eta_M\rho\pi_M\gamma_R^2 + 2\eta_R\rho\pi_R\gamma_M^2\right)}{2\eta_M\eta_R\rho(\delta+\rho)^2} \tag{45}$$

If $f_{Mdn}(\pi_M)$ is a function of π_M, we can find that f_{Mdn} is a quadratic function of π_M. It is very intuitive to derive the second derivative of f_{Mdn} about π_M and we omit the proof that f_{Mdn} is a concave function. Let $f_{Mdn} = 0$, we can obtain the two roots $\pi_{M1}^* = 0$ or $\pi_{M2}^* = \frac{\eta_M\pi_R\gamma_R^2 - (1+2\rho)\eta_R\gamma_M^2\pi_R}{\eta_M\gamma_R^2 + 2\eta_M\rho\gamma_R^2}$. Fig. 3 shows the visualization of f_{Mdn} considering different π_{M2}^*. In Fig. 3, the blue dash areas represent that f_{Mdn} has a positive value and the red dash areas represent f_{Mdn} has a negative value, respectively.

Let $f_{Rdn} = V_R^{sD*}(G) - V_R^{sB*}(G)$, we can derive the expression of f_{Rdn}, which shows as follows:

$$f_{Rdn} = \frac{\theta^2\pi_R\left(\eta_M\pi_M\gamma_R^2 - \eta_R\pi_M\gamma_M^2 + \eta_R\pi_R\gamma_M^2 + 2\eta_M\rho\pi_M\gamma_R^2 + 2\eta_R\rho\pi_R\gamma_M^2\right)}{2\eta_M\eta_R\rho(\delta+\rho)^2} \tag{46}$$

If $f_{Rdn}(\pi_M)$ is a function of π_M, $f_{Rdn}(\pi_M)$ is a linear function. When $f_{Rdn} = 0$, we can get $\pi_{M3}^* = \frac{-(1+2\rho)\eta_R\gamma_M^2\pi_R}{\eta_M\gamma_R^2 - \eta_R\gamma_M^2 + 2\eta_M\rho\gamma_R^2}$. When $\eta_M\gamma_R^2 - \eta_R\gamma_M^2 + 2\eta_M\rho\gamma_R^2 > 0$, f_{Rdn} is a monotone increasing function with $\pi_{M3}^* < 0$. When $\eta_M\gamma_R^2 - \eta_R\gamma_M^2 + 2\eta_M\rho\gamma_R^2 < 0$, f_{Rdn} is a monotone decreasing function with $\pi_{M3}^* > 0$. Fig. 4 shows the visualization of f_{Rdn}. In Fig. 4, the yellow dash areas represent that f_{Rdn} has a positive value and the green dash areas represent f_{Rdn} has a negative value, respectively.

As mentioned above, there are two conditions for f_{Mdn} and f_{Rdn} with given input parameters. Hence, there are four combinations between f_{Mdn} and f_{Rdn}. We explore the conditions of non-negative profits obtained by manufacturer and dealership by analyzing each combination between f_{Mdn} and f_{Rdn}.

Fig. 5 shows the combination between condition I of f_{Mdn} and condition I of f_{Rdn}. From Fig. 5, we can conclude that when $\pi_{M3}^* < \pi_{M2}^* < 0 < \pi_M$ or $\pi_{M2}^* < \pi_{M3}^* < 0 < \pi_M$ the profits of the manufacturer and dealership are always non-negative. Fig. 6 shows the combination between condition I of f_{Mdn} and condition II of f_{Rdn}. From Fig. 6, we can conclude that when $\pi_{M2}^* < 0 < \pi_M < \pi_{M3}^*$ the profits of the manufacturer and dealership are always non-negative.

Fig. 7 shows the combination between condition II of f_{Mdn} and condition I of f_{Rdn}. From Fig. 7, we can conclude that when $\pi_{M3}^* < 0 < \pi_{M2}^* < \pi_M$ the profits of the manufacturer and dealership are always non-negative. Fig. 8 shows the combination between condition II of f_{Mdn} and condition II of f_{Rdn}. From Fig. 8(a), we find that there does not exit a non-negative π_M ensuring that the profits of the manufacturer and dealership are always non-negative simultaneously. From Fig. 8(b), we can conclude that when $0 < \pi_{M2}^* < \pi_M < \pi_{M3}^*$ the profits of the manufacturer and dealership are always non-negative.

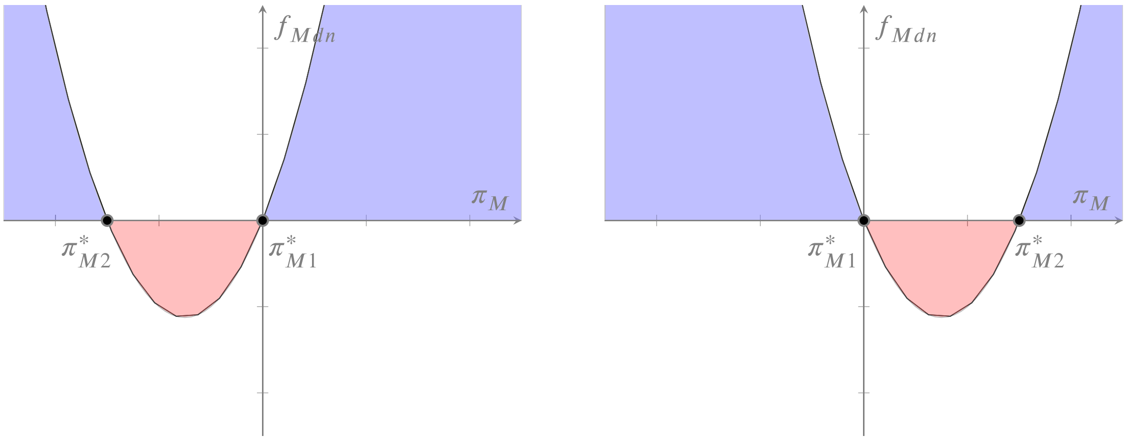
Proposition 7. *When one of the following in-equations is hold, which shows in Table 4, the TWC contract is feasible.*

It can be seen from Proposition 7 that the TWC contract has more complex conditions than the OWC contract. When the TWC contract cannot be established, the OWC contract can be adopted to coordinate the actions of the manufacturer and the dealership.

Next, we analyze the transformation condition from the OWC contract to TWC contract. Let $f_{Mds} = V_M^{sD*}(G) - V_M^{sC*}(G) = \frac{\theta^2\pi_R\left(4\eta_R\pi_M\gamma_M^2(1+2\rho) - \eta_M\pi_R\gamma_R^2 + 4\eta_M\rho\pi_M\gamma_R^2\right)}{8\eta_M\eta_R\rho(\delta+\rho)^2}$. We can find that $f_{Mds}(\pi_M)$ is a monotone increasing function of π_M. Let π_{M4}^* denotes the root of $f_{Mds}(\pi_M) = 0$ and we can derive that $\pi_{M4}^* = \frac{\eta_M\gamma_R^2\pi_R}{4\eta_R\gamma_M^2 + 4\eta_M\rho\gamma_R^2 + 8\eta_R\rho\gamma_M^2}$. When$\pi_M > \pi_{M4}^*$, $f_{Mds} > 0$ is hold. The manufacturer will get higher profits with TWC contract compared with OWC. When $\pi_M < \pi_{M4}^*$, $f_{Mds} < 0$, the manufacturer will get higher profits with OWC contract. From Proposition 6, we know the condition of OWC contract is that $\pi_M > \frac{\pi_R}{2}$. Therefore, considering the conditions for the two type of contracts, if $\pi_M > \pi_{M4}^*$, the manufacturer will adopt the TWC contract. If $1/2\pi_R < \pi_M < \pi_{M4}^*$, the manufacturer will adopt the OWC contract. Finally, we summarize the transformation condition from the OWC contract to TWC contract as Proposition 8.

Proposition 8. *From the perspective of the manufacturer, if the TWC contract is established and $\pi_M > \pi_{M4}^*$, then the manufacturer will prefer the TWC; if $\frac{\pi_R}{2} < \pi_M < \pi_{M4}^*$, then the manufacturer will prefer OWC contract. The difference between the TWC contract and the OWC contract in improving the manufacturer's profits is $f_{Mds} = \frac{\theta^2\pi_R\left(4\eta_R\pi_M\gamma_M^2(1+2\rho) - \eta_M\pi_R\gamma_R^2 + 4\eta_M\rho\pi_M\gamma_R^2\right)}{8\eta_M\eta_R\rho(\delta+\rho)^2}$.*

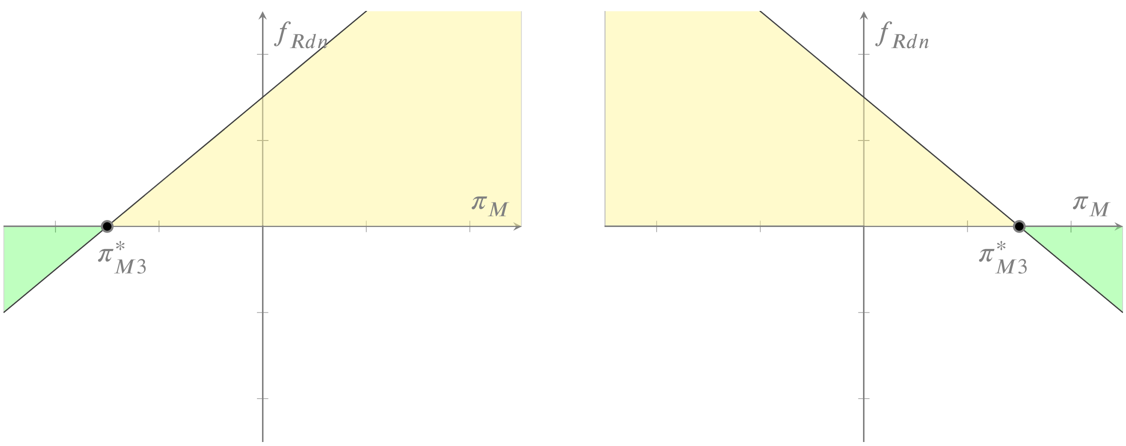
Proposition 8 presents the conditions of contract selection between the OWC contract and TWC contract, which helps the manufacturer to make decisions when designing a contract with the dealership. When the marginal profit of the upstream enterprise is large enough, the two-way cost-sharing contract can obtain better results than the one-way cost-sharing contract. The enterprise needs to choose the contract according to the actual situation.



(a) Condition I of f_{Mdn} with $\pi^*_{M2} < 0$.

(b) Condition II of f_{Mdn} with $\pi^*_{M2} > 0$.

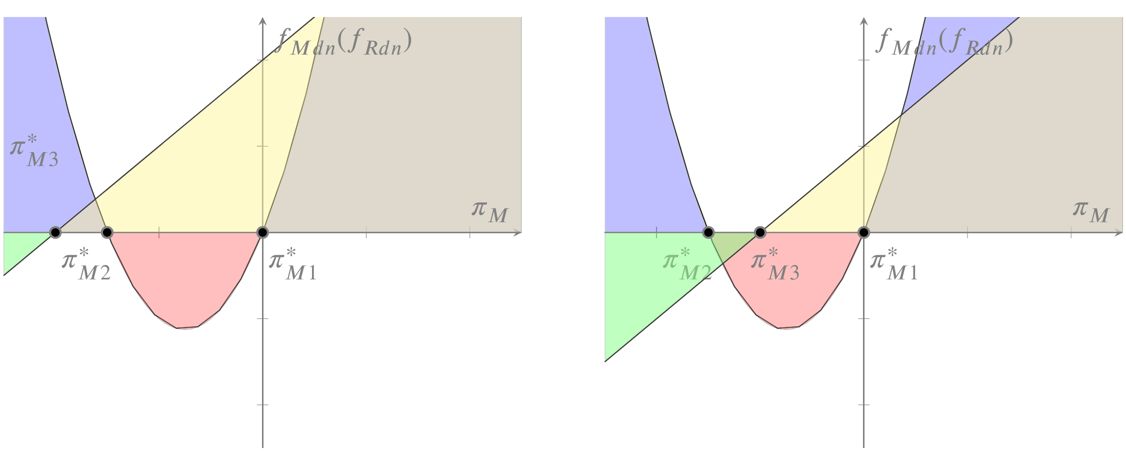
Fig. 3. Visualization of f_{Mdn} with condition I and II ($\pi^*_{M2} < 0$ or $\pi^*_{M2} > 0$).



(a) Condition I of f_{Rdn} with $\eta_M\gamma_R^2 - \eta_R\gamma_M^2 + 2\eta_M\rho\gamma_R^2 > 0$.

(b) Condition II of f_{Rdn} with $\eta_M\gamma_R^2 - \eta_R\gamma_M^2 + 2\eta_M\rho\gamma_R^2 > 0$

Fig. 4. Visualization of f_{Rdn} with condition I and II ($\eta_M\gamma_R^2 - \eta_R\gamma_M^2 + 2\eta_M\rho\gamma_R^2 < 0$ or $\eta_M\gamma_R^2 - \eta_R\gamma_M^2 + 2\eta_M\rho\gamma_R^2 > 0$).



(a) $\pi^*_{M3} < \pi^*_{M2}$

(b) $\pi^*_{M3} > \pi^*_{M2}$

Fig. 5. Case I of f_{Mdn} and case I of f_{Rdn}.

Table 4
Conditions for f_{Mdn} and f_{Rdn} are not non-negative simultaneously.

		f_{Mdn} Condition I	f_{Mdn} Condition II
f_{Rdn}	Condition I	$\pi^*_{M3} < \pi^*_{M2} < 0 < \pi_M$ or $\pi^*_{M2} < \pi^*_{M3} < 0 < \pi_M$	$\pi^*_{M2} < 0 < \pi_M < \pi^*_{M3}$
	Condition II	$\pi^*_{M3} < 0 < \pi^*_{M2} < \pi_M$	$0 < \pi^*_{M2} < \pi_M < \pi^*_{M3}$

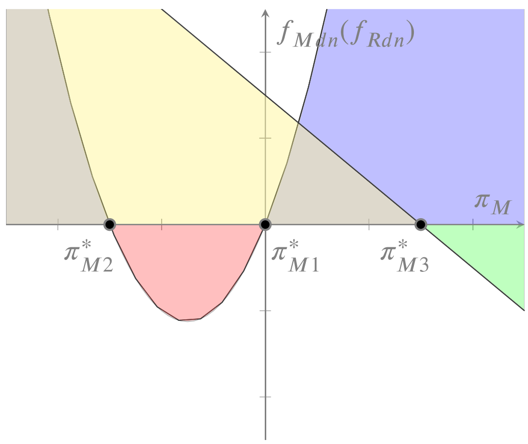


Fig. 6. Case I of f_{Mdn} and case II of f_{Rdn}.

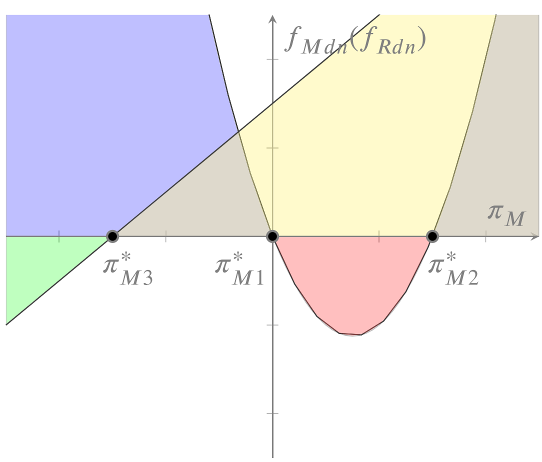


Fig. 7. Case II of f_{Mdn} and case I of f_{Rdn}.

Determining the contract is essential for a manufacturer to sell low-carbon products. In this paper, four different contracts are given for the manufacturer and dealership. Usually, different manufacturers and different dealerships will have different cost coefficients. Determine the contact between a manufacturer and a dealership depending on the participants' parameters (such as cost coefficient) and the contract type. The above propositions give the conditions for each contract and also the conditions for selecting a contract. The above propositions could directly help the manufacturer to determine a contract according to the parameter of the manufacturer and the dealership.

6. Numerical analysis and managerial insights

This section conducts numerical analyses to examine the profit variation of the manufacturer with time-varying and analyze the sensitivity of marginal profits. Finally, some managerial insights are presented.

6.1. Improvement of the manufacturer's profit by Case C and Case D

f_{Mds} provided by 8 defines the profit gaps obtained by the manufacturer in Case C and Case D, respectively, which is a nonlinear function influenced by many factors including η_M, η_R, γ_M, γ_R, ρ, π_M, and π_R. It is difficult to directly derive the variation tendency of f_{Mds} for changing these parameters. Hence, in this paper, we conduct two group of experiments by varying η_M, ρ and γ_R, ρ, respectively. The visualized f_{Mds} shows in Fig. 9. Analyzing the f_{Mds} gives the manufacturer options for choosing OWC contract or TWC contract. When the f_{Mds} is a positive value, manufacturer prefers TWC contract. Otherwise, the manufacturer will adopt the OWC contract.

To make fair comparison, the parameter used in the two group of experiments are same, which defines as follows: $\gamma_M = 2$, $\delta = 1$, $a = 5$, $\pi_M = 1.5$, $\pi_R = 2$, $\theta = 3$, $\eta_R = 12$, and $G_0 = 0$. In the first group of experiment, $\gamma_R = 1.5$. In the second group of experiment, $\eta_M = 15$. The experimental results of the two group of experiments shows in Fig. 9(a) and 9(b), respectively.

From Fig. 9(a), we can conclude that when ρ value is very small, the difference in the manufacturer's profit improvement is mostly negative. When η_M equals 50, the difference in the manufacturer's profit is positive. Therefore, the manufacturer will choose the OWC contract. With the discount rate ρincreasing, the difference in profit improvement is mostly positive, and the manufacturer will choose the TWC contract.

As can be seen from Fig. 9(b), when ρ is very small, the difference in the manufacturer's profit improvement is mostly negative. When γ_R is also very small, the difference in profit improvement is positive. Therefore, when ρ is very small, the manufacturer prefers the OWC contract. With the increase in discount rate ρ, the manufacturer adopts the TWC contract.

The above experiments conclude that the contract design between the manufacturer and the dealership is very complex. The selection between the OWC contract and the TWC contract will be affected by many parameters.

6.2. Changes in manufacturer's and dealership's profits over time

This section discusses the variation of profits for the manufacturer and the dealership over time for OWC contract and TWC contract, respectively.

When $\gamma_M = 2$, $\gamma_R = 1.5$, $\delta = 1$, $a = 5$, $\pi_M = 6$, $\pi_R = 5$, $\theta = 3$, $\eta_M = 15$, $\eta_R = 12$, $G_0 = 0$, $\rho = 0.9$, we conclude that the manufacturer will choose the TWC contract from Proposition 8. When adopting the TWC contract, the variation of the manufacturer's and the dealership's profits over time are shown in Fig. 10a and 10b, respectively.

From Fig. 10, when t is smaller than 1, the profit gaps of the manufacturer and the dealership among the three contract cases are minor. As time increases, the gaps are becoming large and large. Finally, the profits become stable. From Fig. 10, we can conclude that the TWC contract always outperforms the OWC contract. The TWC contract is a win–win contract that both the manufacturer and the dealership prefer to this contract.

When $\gamma_M = 2$, $\gamma_R = 1.5$, $\delta = 1$, $a = 5$, $\pi_M = 1.5$, $\pi_R = 2$, $\theta = 3$, $\eta_M = 150$, $\eta_R = 12$, $G_0 = 0$, $\rho = 0.1$, the manufacturer will choose the OWC contract. At this point, the changes of the manufacturer's and the dealership's profits over time are shown in Fig. 11a and 11b, respectively.

From Fig. 11a, we conclude that the profit of the manufacturer obtained under the OWC contract always outperforms the other two contracts. From Fig. 11b, the profit of the dealership obtained under the TWC contract always outperforms the other two contracts. Even though the manufacturer offers the OWC contract, the dealership still accepts this contract, in which the dealership's profit is still better than

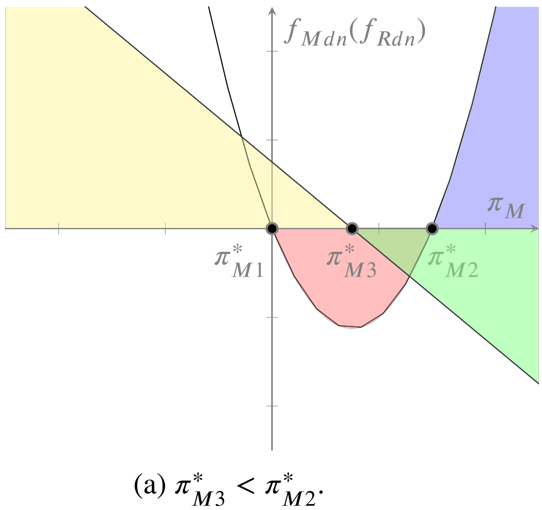


Fig. 8. Case II of f_{Mdn} and case II of f_{Rdn}.

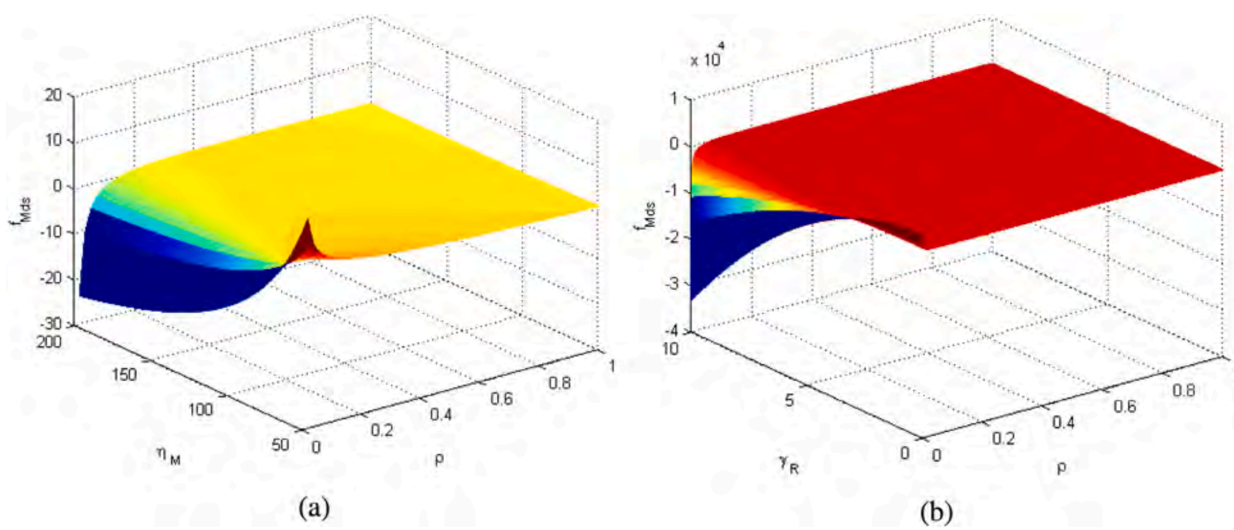


Fig. 9. (a) The profit gaps of the manufacturer between Case C and Case D with varying of η_M and ρ; (b) The profit gaps of the manufacturer between Case C and Case D with varying of γ_R and ρ.

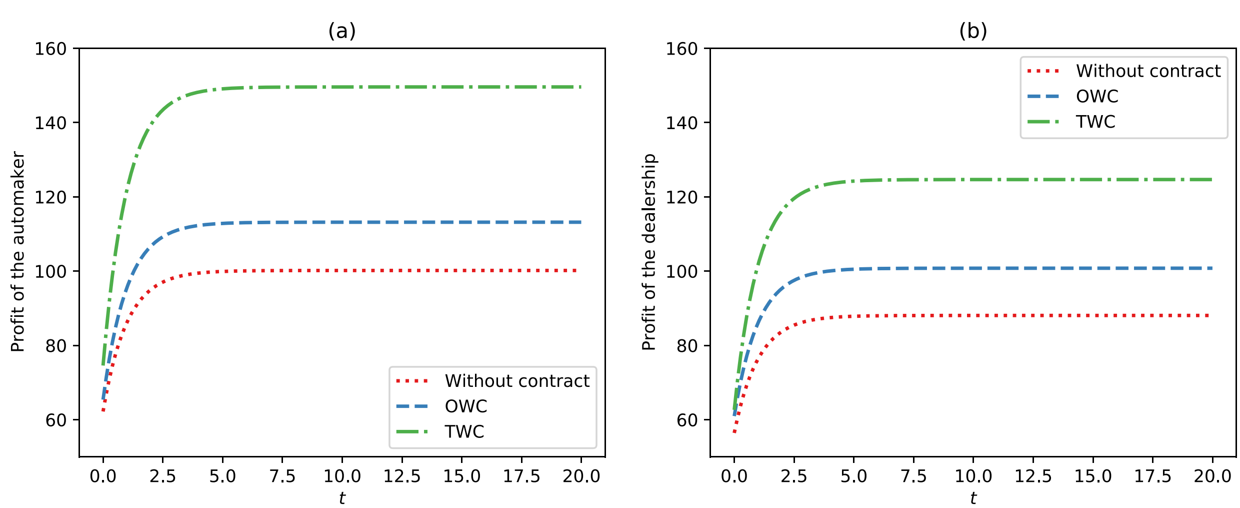


Fig. 10. (a) Change of the manufacturer's profits when TWC contract is selected. (b) Change of the dealership's profits when TWC contract is selected.

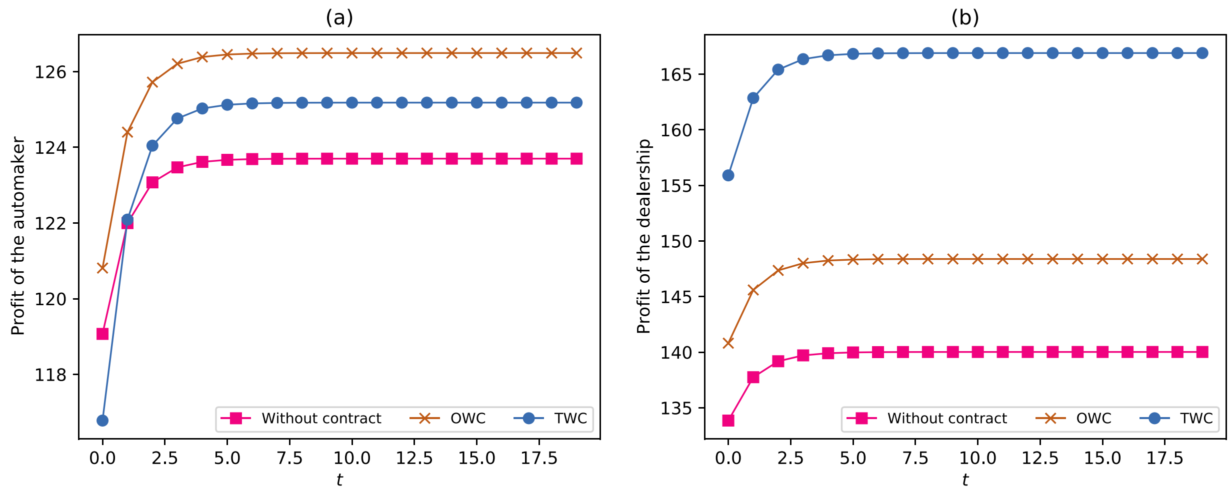


Fig. 11. (a) Change of the manufacturer's profits when OWC contract is selected. (b) Change of the dealership's profits when OWC contract is selected.

Table 5
Parameter setting for sensitive analysis of π_M and π_R considering manufacturer's and dealership's profit.

Parameters	Value	Parameters	Value	Parameters	Value
γ_M	2	γ_R	1.5	ρ	0.1
δ	1	a	5	η_R	12
θ	3	η_M	15	G_0	0

that without a cost-sharing contract. From the experimental results shown in Fig. 11, we find that the dealership will prefer the TWC contract than the OWC contract when the manufacturer offers the OWC contract. If two manufacturers offer an OWC contract and TWC contract to a dealership simultaneously, the dealership will cooperate with the manufacturer who provides the TWC contract. When the value of participation is different, the profits realized by the upstream and downstream through the two contracts are different. Enterprises should fully consider various factors when choosing contracts and find a contract suitable for both parties to cooperate.

6.3. Sensitive analysis of π_M and π_R for manufacturer's and dealership's profit

As mentioned before, the TWC contract can coordinate the actions of the manufacturer and dealership. The manufacturer prefers TWC contract when ρ is not very small. π_M and π_R are very two important parameters that affects the manufacturer and dealership choosing contract. Therefore, in this subsection, we focuses on the sensitive analysis of π_M and π_R considering the manufacturer's and the dealership's profits.

Table 5 shows parameter setting for sensitive analysis of π_M and π_R considering manufacturer's and dealership's profit. The sensitive analysis experimental results for the manufacturer and dealership's profit shows in Figs. 12 and 13, respectively. Figs. 12(a) and 12(b) shows the manufacturer's profit and its contour of the manufacturer's profit function with varying π_M and π_R, respectively. As seen from Fig. 12(a), the profits of the manufacturer increase with π_M and π_R increasing. Fig. 12(b) shows that π_M is more sensitive than π_R in terms of the manufacturer's profit.

Figs. 13(a) and 13(b) shows the dealership's profit and its contour of the dealership's profit function with varying π_M and π_R, respectively. From Fig. 13(a), we can conclude that the dealership's profits increase with π_M and π_R increasing. Fig. 13(b) shows that π_R is more sensitive than π_M in terms of the dealership's profit.

6.4. Managerial insights

This paper investigates collaboration and coordination between the manufacturer and dealership through a cost-sharing contract to promote selling low-carbon production. Some interesting managerial implications from the model analysis and numerical experiments are summarized as follows: (1) The manufacturer bears the low-carbon problem promotion cost in the previous classical coordination contract. The cost-sharing contract stimulates the dealership to participate in the low-carbon problem promotion by sharing parts of the low-carbon problem promotion cost. Cost-sharing contracts have better performance than the traditional dealership model without cost-sharing. Both the manufacturer and the dealership could be benefited from the cost-sharing contract. This paper gives a theoretical and policy basis for manufacturers and dealerships to cooperate when they design contracts. (2) The traditional contract between the manufacturer and the dealership belongs to the Lump-Sum payment contract. The contract fee is paid in one single payment instead of broken up into installments between the manufacturer and dealership without considering the dynamic change of low carbon reputation of low-carbon production. This paper gives another option for the manufacturer when designing the contract. (3) This paper concludes the conditions of choosing an OWC contract or TWC contract, which could help the manufacturer to make the decision for designing contract types. From the proposed propositions, we can find the conditions for OWC and TWC, which could help the manufacturer determine the types of contracts. (4) Besides, we also extended the single-channel marketing with a cost-sharing contract to the dual-channel marketing with a cost-sharing contract. The dual-channel marketing is also analyzed by considering one-way cost-sharing and two-way cost-sharing contract design.

7. Extended models

In reality, many manufacturers are using dual channel marketing (Batarfi, Jaber, & Zanoni, 2016; Ryan, Sun, & Zhao, 2012). The Groupe PSA, the ninth-largest manufacturer in the world, started selling cars online. Such as, in September 2021, the Peugeot sold the compact SUV (2008 THE ONE) online in China.

In this section, this paper analyzes the cooperation between the manufacturer and the dealership under dual channel marketing, which consists of a direct retail channel and an indirect retail channel. Under the direct retail channel, the manufacturer directly seals a proportion of low-carbon productions to the customers. Under the indirect retail channel, the manufacturer directly seals it is certain low-carbon productions to the dealership. A parameter μ is defined as the competition

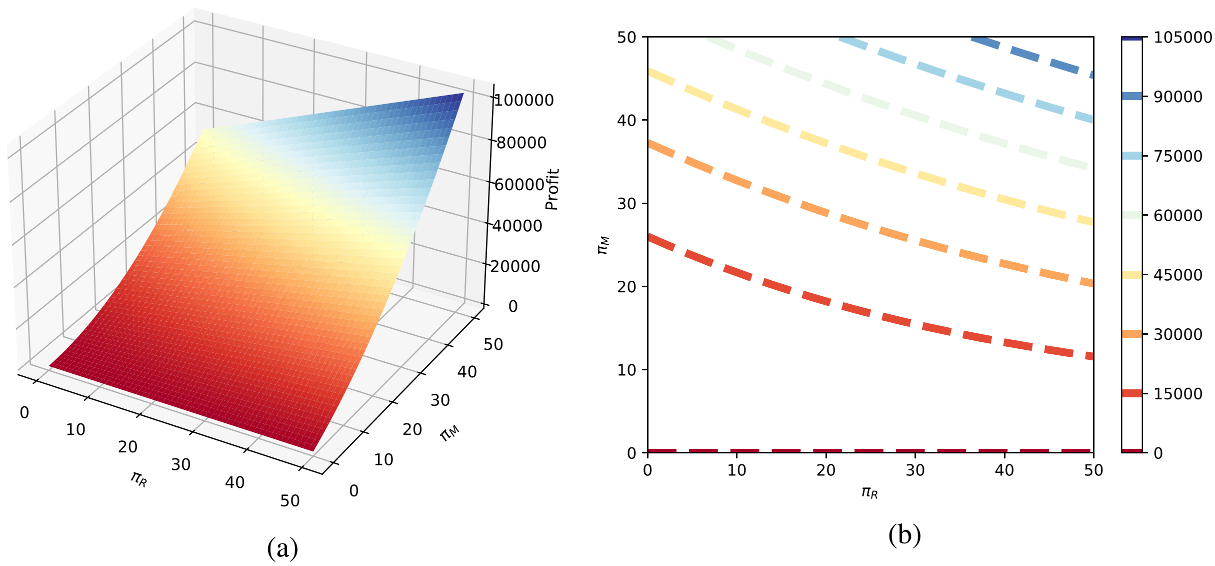


Fig. 12. (a) The manufacturer's profit with varying π_M and π_R. (b) Contour of the manufacturer's profit function with varying π_M and π_R.

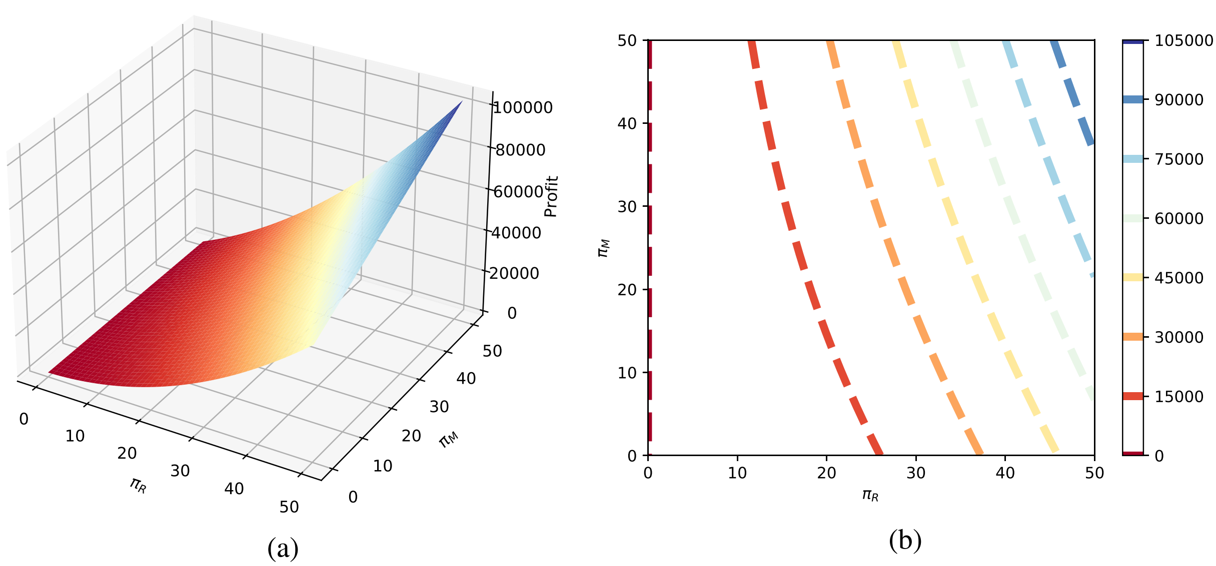


Fig. 13. (a) The dealership's profit with varying π_M and π_R. (b) Contour of the Dealership's profit function with varying π_M and π_R.

coefficient between the two channels. The demand functions for the direct retail channel and the indirect retail channel show in Eqs. (47) and (48), respectively.

$$Q_d(t) = \mu a + \theta G(t) \tag{47}$$

$$Q_r(t) = (1-\mu)a + \theta G(t) \tag{48}$$

7.1. Centralized model under dual-channel

Under the centralized dual-channel model, the objective is maximizing the total profit of the manufacturer and the dealership, which is defined as follows.

$$J_{SC}^{dA} = \max_{E_M, E_R} \int_0^{\infty} e^{-\rho t} \left\{ \pi_M \left(Q_d + Q_r\right) + \pi_R Q_r - C_M\left(E_M\right) - C_R\left(E_R\right) \right\} dt \tag{49}$$

The objective function J_{SC}^{dA} can be solved by using the same method for Lemma 1. After solving the above objective function, we can conclude Lemma 5, which is presented as follow.

Lemma 5. *For the case of a centralized decision model with dual-channel, the equilibrium, optimal trajectory of low carbon reputation, and the optimal profit for the manufacturer and the dealership are presented as follows:*

(i) The equilibrium strategies of the manufacturer and the dealership are $E_M^{dA^*} = \frac{\gamma_M \theta(2\pi_M + \pi_R)}{\eta_M(\rho+\delta)}$ *and* $E_R^{dA^*} = \frac{\gamma_R \theta(\pi_R + 2\pi_M)}{\eta_R(\rho+\delta)}$

(ii) The optimal trajectory of low carbon reputation is $G^{dA^*}(t) = \left(G_0 + \frac{B^{dA}}{A^{dA}}\right) e^{At} - \frac{B^{dA}}{A^{dA}}$

(iii) The optimal profit of the manufacturer and the dealership is $J_{SC}^{dD^*} = e^{-\rho t}\left(a_3^{dA^*} G + b_3^{dA^*}\right)$, *where* $A^{dA} = -\delta$, $B^{dA} = \gamma_M E_M^{dA^*} + \gamma_R E_R^{dA^*}$, $a_3^{dA^*} = \frac{2\pi_M\theta + \pi_R\theta}{\rho+\delta}$ *and* $b_3^{dA^*} = \frac{(\pi_M+\pi_R)a - a\mu\pi_R}{\rho} + \frac{\gamma_M^2\theta(\pi_M+\pi_R)a_3^{dA^*}}{2\eta_M\rho} + \frac{\gamma_R^2\theta(\pi_M+\pi_R)a_3^{dA^*}}{2\eta_R\rho}$.

7.2. Decentralized model without cost-sharing under dual-channel

In this subsection, the manufacturer and the dealership maximize their profit under the decentralized dual-channel without a cost-sharing contract. The objective functions of the manufacturer and the dealership are show in Eqs. (50) and (51), respectively.

$$J_M^{dB} = \max_{E_M} \int_0^{\infty} e^{-\rho t} \left\{ \pi_M \left(Q_d + Q_r\right) - C_M\left(E_M(t)\right) \right\} dt \tag{50}$$

$$J_R^{dB} = \max_{E_R} \int_0^{\infty} e^{-\rho t} \left\{ \pi_R Q_r - C_R(E(t)) \right\} dt \tag{51}$$

The objective function J_M^{dB} and J_R^{dB} can be solved by using the same method for Lemma 2. After solving the above objective function, we can conclude Lemma 6, which is presented as follow.

Lemma 6. *For the case of a decentralized decision model without cost-sharing under dual-channel, the equilibrium, optimal trajectory of low carbon reputation, and the optimal profit for the manufacturer and the dealership are presented as follows:*

(i) The equilibrium strategies of the manufacturer and the dealership are $E_M^{dB^*} = \frac{2\gamma_M \theta \pi_M}{\eta_M(\rho+\delta)}$ *and* ${E_R}^{dB^*} = \frac{\gamma_R \theta \pi_R}{\eta_R(\rho+\delta)}$.

(ii) The optimal trajectory of low carbon reputation is $G^{dB^*}(t) = \left(G_0 + \frac{B^{dB}}{A^{dB}}\right) e^{At} - \frac{B^{dB}}{A^{dB}}$, *where* $A^{dB} = -\delta, B^{dB} = \gamma_M E_M^{dB^*} + \gamma_R E_R^{dB^*}$.

(iii) The optimal profit of the manufacturer and the dealership are $J_M^{dB^*} = e^{-\rho t}\left(a_1^{dB^*} G + b_1^{dB^*}\right)$ *and* $J_R^{dB^*} = e^{-\rho t}\left(a_2^{dB^*} G + b_2^{dB^*}\right)$, *respectively, where* $a_1^{dB^*} = \frac{2\pi_M \theta}{\rho+\delta}$, $a_2^{dB^*} = \frac{\pi_R \theta}{\rho+\delta}$, $b_1^{dB^*} = \frac{\pi_M a}{\rho} + \frac{\gamma_M^2 \left[a_1^{B^*}\right]^2}{2\eta_M \rho} + \frac{\gamma_R^2 \theta \pi_M a_2^{B^*}}{\eta_R \rho(\rho+\delta)}$ *and* $b_2^{dB^*} = \frac{\pi_R(1-u)a}{\rho} + \frac{\gamma_R^2 \left[a_R^{B^*}\right]^2}{2\eta_R \rho} + \frac{\gamma_M^2 \theta \pi_R a_1^{B^*}}{\eta_M \rho(\rho+\delta)}$.

7.3. One-way cost-sharing under decentralized dual-channel

In this subsection, the manufacturer and the dealership maximize their profit under the decentralized dual-channel with a one-way cost-sharing contract. The objective functions of the manufacturer and the dealership are show in Eqs. (52) and (53), respectively.

$$J_M^{dC} = \max_{E_M, X} \int_0^{\infty} e^{-\rho t} \left\{ \pi_M \left(Q_d + Q_r\right) - C_M\left(E_M\right) - X C_R\left(E_R\right) \right\} dt \tag{52}$$

$$J_R^{dC} = \max_{E_R} \int_0^{\infty} e^{-\rho t} \left\{ \pi_R Q_r - (1-X) C_R\left(E_R\right) \right\} dt \tag{53}$$

The objective function J_M^{dC} and J_R^{dC} can be solved by using the same method for Lemma 2. After solving the above objective function, we can conclude Lemma 6, which is presented as follow.

Lemma 7. *For the case of a decentralized decision model without cost-sharing under dual-channel, the equilibrium, optimal trajectory of low carbon reputation, and the optimal profit for the manufacturer and the dealership are presented as follows:*

(i) The equilibrium strategies of the manufacturer and the dealership are $E_M^{aC^*} = \frac{2\gamma_M \theta \pi_M}{\eta_M(\rho+\delta)}$, ${E_R}^{dC^*} = \frac{\gamma_R \theta(\pi_R + 4\pi_M)}{2\eta_R(\rho+\delta)}$ *where* $X^{dC^*} = \frac{4\pi_M - \pi_R}{4\pi_M + \pi_R}$.

(ii) The optimal trajectory of low carbon reputation is $G^{dC^*}(t) = \left(G_0 + \frac{B^{dC}}{A^{dC}}\right) e^{At} - \frac{B^{dC}}{A^{dC}}$, *where* $A^{dC} = -\delta, B^{dC} = \gamma_M E_M^{dC^*} + \gamma_R {E_R}^{dC^*}$.

(iii) The optimal profit of the manufacturer and the dealership are $J_M^{dC^*} = e^{-\rho t}\left(a_1^{dC^*} G + b_1^{dC^*}\right)$ *and* $J_R^{dC^*} = e^{-\rho t}\left(a_2^{dC^*} G + b_2^{dC^*}\right)$, *respectively, where* $a_1^{dC^*} = \frac{2\pi_M \theta}{\rho+\delta}$, $a_2^{dC^*} = \frac{\pi_R \theta}{\rho+\delta}$, $b_1^{dC^*} = \frac{\pi_M a}{\rho} + \frac{{\gamma_M}^2 \left[a_1^{C^*}\right]^2}{2\eta_M \rho} + \frac{{\gamma_R}^2 \left(2a_1^{C^*} + a_2^{C^*}\right)^2}{8\eta_R \rho}$ *and* $b_2^{dC^*} = \frac{\pi_R a(1-u)}{\rho} + \frac{{\gamma_R}^2 a_2^{C^*}\left(2a_1^{C^*} + a_2^{C^*}\right)}{4\eta_R \rho} + \frac{{\gamma_M}^2 a_1^{C^*} a_2^{C^*}}{\eta_M \rho}$.

7.4. Two-way cost-sharing under decentralized dual-channel

In this subsection, the manufacturer and the dealership maximize their profit under the decentralized dual-channel with a one-way cost-sharing contract. The objective functions of the manufacturer and the dealership are show in Eqs. (54) and (55), respectively.

$$J_M^{dD} = \max_{X,Y} \int_0^{\infty} e^{-\rho t} \left\{ \pi_M \left(Q_d + Q_r\right) - (1-Y) C_M\left(E_M\right) - X C_R\left(E_R\right) \right\} dt \tag{54}$$

$$J_R^{dD} = \max_{E_M, E_R} \int_0^{\infty} e^{-\rho t} \left\{ \pi_R Q_r - Y C_M\left(E_M\right) - (1-X) C\left(E_R\right) \right\} dt \tag{55}$$

The objective function J_M^{dD} and J_R^{dD} can be solved by using the same method for Lemma 2. After solving the above objective function, we can conclude Lemma 6, which is presented as follow.

Lemma 8. *For the case of decentralized decision model without cost-sharing under dual-channel, the equilibrium, optimal trajectory of low carbon reputation and the optimal profit for the manufacture and the dealership are presented as follows:*

(i) The equilibrium strategies of the manufacturer and the dealership are $E_M^{dD^*} = \frac{\gamma_M \theta(4\pi_M + \pi_R)}{2\eta_M(\rho+\delta)}$ *and* $E_R^{dD^*} = \frac{\gamma_R \theta(\pi_R + 2\pi_M)}{2\eta_R(\rho+\delta)}$, *where* $X^{dD^*} = \frac{2\pi_M - \pi_R}{2\pi_M + \pi_R}$ *and* $Y^{dD^*} = \frac{2\pi_R}{\pi_R + 4\pi_M}$.

(ii) The optimal trajectory of low carbon reputation is $G^{dD^*}(t) = \left(G_0 + \frac{B^{dD}}{A^{dD}}\right) e^{At} - \frac{B^{dD}}{A^{dD}}$, *where* $A^{dD} = -\delta, B^{dD} = \gamma_M {E_M}^{dD^*} + \gamma_R E_R^{dD^2}$.

(iii) The optimal profit of the manufacturer and the dealership are $J_M^{dD^*} = e^{-\rho t}\left(a_1^{dD^*} G + b_1^{dD^*}\right)$ *and* $J_R^{dD^*} = e^{-\rho t}\left(a_2^{dD^*} G + b_2^{dD^*}\right)$, *respectively, where* $a_1^{dD^*} = \frac{2\pi_M \theta}{\rho+\delta}$, $a_2^{dD^*} = \frac{\pi_R \theta}{\rho+\delta}$, $b_1^{dD^*} = \frac{\pi_M a}{\rho} + \frac{\gamma_M^2 \left(2a_1^{D^*} + a_2^{D^*}\right)^2}{8\eta_M \rho} + \frac{4\gamma_R^2 a_1^{D^*}\left(a_1^{D^*} + a_2^{D^*}\right) - \gamma_R^2\left(\left[a_1^{D^*}\right]^2 - \left[a_2^{D^*}\right]^2\right)}{8\eta_R \rho}$, *and* $b_2^{dD^*} = \frac{\pi_R(1-u)a}{\rho} + \frac{\gamma_M^2\left(4\left[a_1^{D^*}\right]^2 - \left[a_2^{D^*}\right]^2\right)}{4\eta_M \rho} + \frac{\gamma_R^2 a_1^{D^*} a_2^{D^*}}{4\eta_R \rho}$.

8. Concluding remarks and areas for future research

To enhance the collaboration between the manufacturer and the dealership by reducing the exogenous risk incurred by the low carbon reputation of low-carbon products, this paper introduces several differential game models in which the manufacturer is a leader who takes the initiative, and the dealership is a follower to work with the manufacturer for low-carbon products. This paper introduced two cost-sharing contracts, including a one-way cost-sharing contract (OWC) and a two-way cost-sharing contract (TWC). We also analyzed centralized decision and decentralized decision cases between the manufacturer and the dealership.

This paper focuses on analyzing the conditions for cost-sharing contract design between the manufacturer and the dealership. Several essential theories and properties were proposed for different contract cases, which helps to analyze the manufacturer's contract selection conditions. Significant findings are summarized as follows: (1) The OWC contract can enhance the dealership's efforts in a low-carbon promotion. This contract option, however, cannot change the efforts of the manufacturer to reduce its emissions. It can also improve the manufacturer and the dealership to achieve a Pareto improvement but cannot coordinate the actions of the manufacturer and the dealership. (2) The TWC contract can enhance both the manufacturer's and the dealership's efforts and simultaneously coordinate them. (3) The OWC contract and TWC contract have their conditions to be established. Only when the conditions are satisfied can the appropriate contract option be selected. (4) The manufacturer will not always determine the OWC contract or the TWC contract but will choose one of the two options based on improving its profits. (5) Managerial insights were present to help the manufacturer make decisions.

The models considered in this paper do not include government subsidy, carbon tax, carbon trading policy, and other environmental regulations on the low-carbon actions of the manufacturer and the dealership, which could be studied in future research. There are also several avenues for future studies. (1) The proposed contract models have tremendous potential for applying to other topics to manage their emission reduction and optimize their operation decisions. (2) The proposed contract game models can be extended to investigate the contract design between a manufacturer and multiple dealerships. (3) The information asymmetry between the manufacturer and the dealership can be extended. (4) Uncertain of Cost and marginal profit will be another direction to analyze the contract design between the manufacturer and the dealership.

CRediT authorship contribution statement

Chunqiu Xu: Conceptualization, Methodology, Funding acquisition, Writing – original draft. **Yu Jing:** Experiment, Data collection, Writing. **Bo Shen:** Conceptualization, Validation. **Yanjie Zhou:** Conceptualization, Validation, Formal analysis, Writing – review & editing, Project administration. **Qian Qian Zhao:** Conceptualization, Supervision, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Appendix A. Proof of Lemma 1

With the centralized decision model, the optimal profit function of central decision maker can be modeled as follows:

$$J_{sc}^{sA} = \max_{E_M^{sA}, E_R^{sA}} \int_0^{\infty} e^{-\rho t} \left\{ (\pi_M + \pi_R) Q - C_M(E_M^{sA}) - C_R(E_R^{sA}) \right\} dt. \tag{A.1}$$

According to Eq. (A.1), the optimal profit function of the central decision maker at time t can be rewrite as

$$J_{sc}^{sA} = e^{-\rho t} V_{sc}^{sA}(G), \tag{A.2}$$

where $V_{sc}^{sA}(G) = \max_{E_M, E_R} \int_t^{\infty} e^{-\rho(s-t)} \{(\pi_M + \pi_R)Q - C_M(E_M^{sA}) - C_R(E_R^{sA})\} ds$.

Now the HJB equation for central decision maker can be write as

$$\rho V_{sc}^{sA}(G) = \max_{E_M^{sA}, E_R^{sA}} \Big\{ (\pi_M + \pi_R) Q - C_M(E_M^{sA}) - C_R(E_R^{sA}) + V_{sc}^{C'}(G)(\gamma_M E_M^{sA} + \gamma_R E_R^{sA} - \delta G) \Big\}. \tag{A.3}$$

Substituting Eqs. (1), (2) and low-carbon promotion cost function into (A.3) we can obtain that:

$$\rho V_{sc}^{sA}(G) = \max_{E_M^{sA} E_R^{sA}} \Big\{ (\pi_M + \pi_R)[a + \theta G] - \frac{1}{2} \eta_M \left(E_M^{sA}\right)^2 - \frac{1}{2} \eta_R \left(E_R^{sA}\right)^2 + V_{sc}^{c'}(G) \left(\gamma_M E_M^{sA} + \gamma_R E_R^{sA} - \delta G\right) \Big\} \tag{A.4}$$

According to the second derivative and the hessian matrix, we can know that f is a concave function about E_M^{sA} and E_R^{sA}.

$$f = \max_{E_M^{sA} E_R^{sA}} \{(\pi_M + \pi_R)[a + \theta G] - \frac{1}{2} \eta_M (E_M^{sA})^2 - \frac{1}{2} \eta_R (E_R^{sA})^2 + V_{sc}^{C'}(G)(\gamma_M E_M^{sA} + \gamma_R E_R^{sA} - \delta G)\} \tag{A.5}$$

By solving $\partial f / \partial E_M^{sA} = 0$ and $\partial f / \partial E_R^{sA} = 0$, we can get the following results:

$$E_M^{sA} = \frac{\gamma_M V_{sc}^{sA'}(G)}{\eta_M}, E_R^{sA} = \frac{\gamma_R V_{sc}^{sA'}(G)}{\eta_R} \tag{A.6}$$

Substituting (A.6) into (A.4), we can get:

$$\rho V_{sc}^{sA}(G) = [(\pi_M + \pi_R)\theta - V_{sc}^{sA'}(G)\delta] G + \frac{\gamma_M^2 [V_{sc}^{C'}(G)]^2}{2\eta_M} + \frac{\gamma_R{}^2 [V_{sc}^{sA'}(G)]^2}{2\eta_R} + (\pi_M + \pi_R) a \tag{A.7}$$

Taking a cue from Sethi (1983), we conjecture value functions in linear forms as follows:

$$V_{sc}^{sA}(G) = a_3^{sA} G + b_3^{sA}, \tag{A.8}$$

where a_3^{sA} and b_3^{sA} are unknown constants. Substitute (A.8) into (A.7) we can get the equation as follows:

$$\rho(a_3^{sA} G + b_3^{sA}) = [(\pi_M + \pi_R)\theta - \delta a_3^{sA}] G + \frac{(\gamma_M)^2 (a_3^{sA})^2}{2\eta_M} + \frac{(\gamma_R)^2 (a_3^{sA})^2}{2\eta_R} + (\pi_M + \pi_R) a \tag{A.9}$$

Contrasting the two sides of (A.9), the following two equations can be get.

$$\begin{cases} \rho a_3^{sA} = (\pi_M + \pi_R)\theta - \delta a_3^C \\ \rho b_3^{sA} = \frac{(\gamma_M)^2 a_3^{sA\,2}}{2\eta_M} + \frac{(\gamma_R)^2 a_3^{sA\,2}}{2\eta_R} + (\pi_M + \pi_R) a \end{cases} \tag{A.10}$$

Solving the equation system (A.10), we can obtain a_3^{sA*} and b_3^{sA*}, which show as follows:

$$a_3^{sA*} = \frac{(\pi_M + \pi_R)\theta}{\rho + \delta} \tag{A.11}$$

$$b_3^{sA*} = \frac{(\gamma_M)^2 (\pi_M + \pi_R)^2 \theta^2}{2\eta_M \rho(\rho + \delta)} + \frac{(\gamma_R)^2 (\pi_M + \pi_R)^2 \theta^2}{2\eta_R \rho(\rho + \delta)} + \frac{(\pi_M + \pi_R) a}{\rho} \tag{A.12}$$

Substituting a_3^{sA*} and b_3^{sA*} into (A.8), we can get

$$V_{sc}^{sA*}(G) = a_3^{sA*} G + b_3^{sA*} \tag{A.13}$$

Substituting the derivative of (A.13) into (A.6), the optimal strategies for the supply chain are

$$E_M^{sA*} = \frac{\gamma_M (\pi_M + \pi_R)\theta}{\eta_M (\rho + \delta)}, E_R^{sA*} = \frac{\gamma_R (\pi_M + \pi_R)\theta}{\eta_R (\rho + \delta)} \tag{A.14}$$

Substituting (A.14) into Eq. (2), according to the boundary conditions of the state equation, the optimal trajectory (8) of low-carbon reputation can be obtained. Substituting (A.13) into (A.2), we can get the optimal profit function (9) of the supply chain. ■

Appendix B. Proof of Lemma 2

Based on the assumptions, the objective functions of manufacturer and dealership are represented as follows:

$$J_M^{sB} = \max_{E_M^{sB}(t)} \int_0^{\infty} e^{-\rho t} \left\{ \pi_M Q - C_M(E_M^{sB}(t)) \right\} dt \tag{B.1}$$

$$J_R^{sB} = \max_{E_R^{sB}(t)} \int_0^{\infty} e^{-\rho t} \left\{ \pi_R Q - C_R(E_R^{sB}(t)) \right\} dt \tag{B.2}$$

Based on the backward induction method, we first derive the optimal response function of the dealership. The dealership's optimal profit function at time t can be derived as:

$$J_R^{sB*}(G, t) = e^{-\rho t} V_R^{sB}(G), \tag{B.3}$$

where $V_R^{sB}(G)$ equals to $\max_{E_R^{sB}} \int_t^{\infty} e^{-\rho(s-t)} \left\{ \pi_R Q - C_R(E_R^{sB}) \right\} ds$.

Then, the HJB equation of the dealership can be written as

$$\rho V_R^{sB}(G) = \max_{E_R^{sB}} \left\{ \pi_R Q - C_R(E_R^{sB}) + V_R^{sB'}(G)\dot{G} \right\}. \tag{B.4}$$

Substituting (1), (2) and low-carbon promotion cost function into (B.4), we can obtain that,

$$\rho V_R^{sB}(G) = \max_{E_R^{sB}} \left\{ \pi_R [a + \theta G] - \frac{1}{2} \eta_R \left(E_R^{sB}\right)^2 \right.$$

$$+V_R^{sB'}(G)[\gamma_M E_M^{sB}+\gamma_R E_R^{sB}-\delta G]\Big\} \tag{B.5}$$

From the second derivative, we know that the right hand side of (B.5) is a concave function about E_R. Let the first derivative equal to zero, the dealership's response function can be obtained as:

$$E_R=\frac{\gamma_R V_R^{sB'}(G)}{\eta_R} \tag{B.6}$$

The manufacturer's optimal profit function (14) at time t can be denoted as:

$$J_M^{sB^*}(G,t)=e^{-\rho t}V_M^{sB}(G), \tag{B.7}$$

where $V_M^{sB}(G)=\max_{E_M^{sB}}\int_t^\infty e^{-\rho(s-t)}\left\{\pi_M Q-C_M(E_M^{sB})\right\}ds$.

The HJB equation for manufacturer can be expressed as:

$$\rho V_M^{sB}(G)=\max_{E_M}\left\{\pi_M Q-C_M(E_M^{sB})+V_M^{sB'}(G)\dot G\right\} \tag{B.8}$$

Substituting dealership's response function into (B.8), we can get:

$$\rho V_M^{sB}(G)=\max_{E_M^{sB}}\{\pi_M[a+\theta G]-\frac{1}{2}\eta_M\left(E_M^{sB}\right)^2 + V_M^{sB'}(G)[\gamma_M E_M^{sB}+\frac{\gamma_R^2 V_R^{sB'}(G)}{\eta_R}-\delta G]\} \tag{B.9}$$

The right hand side of (B.8) is a concave function about E_M^{sB}. According to the first derivative, the manufacturer's optimal emission efforts can be obtained

$$E_M^{sB}=\frac{\gamma_M V_M^{sB'}(G)}{\eta_M} \tag{B.10}$$

Substituting (B.6) and (B.10) into (B.4) and (B.8), respectively, the HJB equations for manufacturer and dealership can be expressed as

$$\rho V_M^{sB}(G)=\left(\pi_M\theta-\delta V_M^{sB'}(G)\right)G+\frac{(\gamma_M)^2\left(V_M^{sB'}(G)\right)^2}{2\eta_M} + \pi_M a+\frac{(\gamma_R)^2 V_M^{sB'}(G)V_R^{sB'}(G)}{\eta_R} \tag{B.11}$$

$$\rho V_R^{sB}(G)=\left(\pi_R\theta-\delta V_R^{sB'}(G)\right)G+\frac{(\gamma_R)^2\left(V_R^{sB'}(G)\right)^2}{2\eta_R} + \pi_R a+\frac{(\gamma_M)^2 V_M^{sB'}(G)V_R^{sB'}(G)}{\eta_M} \tag{B.12}$$

Taking a cue from Sethi (1983), we conjecture value functions in linear forms as follows:

$$V_M^{sB}(G)=a_1^{sB}G+b_1^{sB}\quad and\quad V_R^{sB}(G)=a_2^{sB}G+b_2^{sB}, \tag{B.13}$$

where $a_1^{sB},b_1^{sB},a_2^{sB},b_2^{sB}$ are constant values.

Substituting (B.13) into (B.11) and (B.12), the HJB equations for manufacturer and dealership can be written as:

$$\rho(a_1^{sB}G+b_1^{sB})=\left(\pi_M\theta-\delta a_1^{sB}\right)G+\frac{(\gamma_M)^2\left(a_1^{sB}\right)^2}{2\eta_M}+\pi_M a+\frac{(\gamma_R)^2 a_1^{sB}a_2^{sB}}{\eta_R} \tag{B.14}$$

$$\rho(a_2^{sB}G+b_2^{sB})=\left(\pi_R\theta-\delta a_2^{sB}\right)G+\frac{(\gamma_R)^2\left(a_2^{sB}\right)^2}{2\eta_R}+\pi_R a+\frac{(\gamma_M)^2 a_1^{sB}a_2^{sB}}{\eta_M} \tag{B.15}$$

Contrasting the two sides of each equation, (B.14) and (B.15) can be written as follows:

$$\begin{cases}\rho a_1^{sB}=\pi_M\theta-\delta a_1^{sB} & , \quad \rho b_1^{sB}=\pi_M a+\frac{(\gamma_M)^2\left(a_1^{sB}\right)^2}{2\eta_M}+\frac{(\gamma_R)^2 a_1^{sB}a_2^{sB}}{\eta_R}\\ \rho a_2^{sB}=\pi_R\theta-\delta a_2^{sB} & , \quad \rho b_2^{sB}=\pi_R a+\frac{(\gamma_R)^2\left(a_2^{sB}\right)^2}{2\eta_R}+\frac{(\gamma_M)^2 a_1^{sB}a_2^{sB}}{\eta_M}\end{cases} \tag{B.16}$$

Solving the above equations, $a_1^{sB*},b_1^{sB*},a_2^{sB*},b_2^{sB*}$ can be obtained as:

$$\begin{cases}a_1^{sB*}=\frac{\pi_M\theta}{\delta+\rho} & , \quad b_1^{sB*}=\frac{\pi_M a}{\rho}+\frac{(\pi_M)^2\theta^2(\gamma_M)^2}{2\eta_M\rho(\delta+\rho)^2}+\frac{\pi_M\pi_R\theta^2(\gamma_R)^2}{\eta_R\rho(\delta+\rho)^2}\\ a_2^{sB*}=\frac{\pi_R\theta}{\delta+\rho} & , \quad b_2^{sB*}=\frac{\pi_R a}{\rho}+\frac{(\pi_R)^2\theta^2(\gamma_R)^2}{2\eta_R\rho(\delta+\rho)^2}+\frac{\pi_R\pi_M\theta^2(\gamma_M)^2}{\eta_M\rho(\delta+\rho)^2}\end{cases} \tag{B.17}$$

Substituting $a_1^{sB*},b_1^{sB*},a_2^{sB*},b_2^{sB*}$ into (B.13), the following functions can be denoted as:

$$V_M^{sB*}(G)=a_1^{sB*}G+b_1^{sB*}\quad and\quad V_R^{sB*}(G)=a_2^{sB*}G+b_2^{sB*} \tag{B.18}$$

Substituting the derivative of (B.18) into (B.6) and (B.10), the optimal solution of the manufacturer and the dealership can be written as follows:

$$E_M^{sB*}=\frac{\gamma_M\pi_M\theta}{\eta_M(\delta+\rho)}\quad and\quad E_R^{sB*}=\frac{\gamma_R\pi_R\theta}{\eta_R(\delta+\rho)} \tag{B.19}$$

According to the boundary conditions and substituting (B.19) into (2), the optimal trajectory of low-carbon reputation can be obtained as (18). In addition, substituting (B.18) into (B.3) and (B.7), we can get the optimal profit function (19) and (20) for the manufacturer and the dealership, respectively. ■

Appendix C. Proof of Lemma 3

The objective functions of manufacturer and dealership are represented as follows:

$$J_M^{sC}=\max_{E_M^{sC},X^{sC}}\int_0^\infty e^{-\rho t}\left\{\pi_M Q-C_M(E_M^{sC})-X^{sC}C_R(E_R^{sC})\right\}dt \tag{C.1}$$

$$J_R^{sC}=\max_{E_R^{sC}}\int_0^\infty e^{-\rho t}\left\{\pi_R Q-(1-X^{sC})C_R(E_R^{sC})\right\}dt \tag{C.2}$$

Similar to the proof of Lemmas 1 and 2, the dealership's response function can be expressed as follows:

$$E_R^{sC}=\frac{\gamma_R V_R'(G)}{\eta_R(1-X^{sC})} \tag{C.3}$$

The manufacturer's optimal profit function at time t is:

$$J_M^{sC*}(G,t) =\max_{E_M^{sC},X^{sC}}\int_t^\infty e^{-\rho s}\left\{\pi_M Q-C_M(E_M^{sC})-X^{sC}C_R(E_R^{sC})\right\}dt \tag{C.4}$$

(C.4) can be rewritten as:

$$J_M^{sC*}(G,t)=e^{-\rho t}V_M(G) \tag{C.5}$$

where $V_M(G)$ can be expressed as:

$$V_M^{sC}(G)=\max_{E_M^{sC},X^{sC}}\int_t^\infty e^{-\rho(s-t)}\left\{\pi_M Q-C_M(E_M^{sC})-XC_R(E_R^{sC})\right\}dt \tag{C.6}$$

The manufacturer's optimal control problem satisfies the following HJB equation:

$$\rho V_M^{sC}(G)=\max_{E_M^{sC},X^{sC}}\left\{\pi_M Q-C_M(E_M)-X^{sC}C_R(E_R^{sC})+V_M^{sC'}(G)\dot G\right\} \tag{C.7}$$

Substituting the dealership's response function (C.3) into Eq. (C.7), we can get

$$\rho V_M^{sC'}(G)=\max_{E_M^{sC'},X^{sC'}}\left\{\begin{array}{l}\pi_M[a+\theta G]-\frac{1}{2}\eta_M E_M^{sC'2}-\frac{1}{2}X^{sC'}\eta_R\left[\frac{\gamma_R V_R^{sC'}(G)}{\eta_R(1-X^{sC'})}\right]^2\\ +V_M^{sC'}(G)\left[\gamma_M E_M+\gamma_R\left[\frac{\gamma_R V_R^{sC'}(G)}{\eta_R(1-X^{sC'})}\right]-\delta G\right]\end{array}\right\} \tag{C.8}$$

According to the second derivative and the hessian matrix, we can know that the right hand side of Eq. (C.8) is a concave function about E_M^{sC} and E_R^{sC}. Therefore, E_M^{sC}, X^{sC} can be got as follows:

$$E_M^{sC}=\frac{\gamma_M V_M^{sC'}(G)}{\eta_M}, X^{sC}=\frac{2V_M^{sC}(G)-V_R^{sC'}(G)}{2V_M^{sC'}(G)+V_R^{sC'}(G)} \tag{C.9}$$

Substituting (E_M^{sC}, X^{sC}) and E_R^{sC}, The HJB equations of manufacturer and dealership are derived as follows:

$$\rho V_M^{sC}(G)=[\pi_M\theta-\delta V_M^{sC'}(G)]G+\frac{[\gamma_M V_M^{sC'}(G)]^2}{2\eta_M} + \pi_M a+\frac{\gamma_R{}^2[2V_M^{sC'}(G)+V_R^{sC'}(G)]^2}{8\eta_R} \tag{C.10}$$

$$V_R^{sC}(G)=[\pi_R\theta-V_R^{sC'}(G)\delta]G+\frac{\gamma_M{}^2 V_R^{sC'}(G)V_M^{sC'}(G)}{\eta_M} + \pi_R a+\frac{\gamma_R{}^2 V_R^{sC'}(G)[2V_M^{sC'}(G)+V_R^{sC'}(G)]}{4\eta_R} \tag{C.11}$$

It is assumed that:

$$V_M^{sC}(G)=a_1^{sC}G+b_1^{sC}, V_R(G)=a_2^{sC}G+b_2^{sC}, \tag{C.12}$$

where $a_1^{sC}, b_1^{sC}, a_2^{sC}, b_2^{sC}$ are unknown constants.

Substitute (C.12) into Eqs. (C.10) and (C.11) we can get

$$\rho(a_1^{sC}G+b_1^{sC})=[\pi_M\theta-\delta a_1^{sC}]G+\frac{[\gamma_M a_1^{sC}]^2}{2\eta_M}+\pi_M a+\frac{\gamma_R^2[2a_1^{sC}+a_2^{sC}]^2}{8\eta_R} \tag{C.13}$$

$$\rho(a_2^{sC}G+b_2^{sC})=[\pi_R\theta-\delta a_2^{sC}]G+\frac{\gamma_M^2 a_2^{sC}a_1^{sC}}{\eta_M}+\pi_R a+\frac{\gamma_R^2 a_2^{sC}[2a_1^{sC}+a_2^{sC}]}{4\eta_R} \tag{C.14}$$

Comparing the similarities at both ends of the equations of (C.13) and (C.14), we can get

$$\begin{cases}\rho a_1^{sC}=\pi_M\theta-\delta a_1^{sC}\\ \rho b_1^{sC}=\frac{[\gamma_M a_1^{sC}]^2}{2\eta_M}+\pi_M a+\frac{\gamma_R^2[2a_1^{sC}+a_2^{sC}]^2}{8\eta_R}\\ \rho a_2^{sC}=\pi_R\theta-\delta a_2^{sC}\\ \rho b_2^{sC}=\frac{\gamma_M^2 a_2^{sC}a_1^{sC}}{\eta_M}+\pi_R a+\frac{\gamma_R^2 a_2^{sC}[2a_1^{sC}+a_2^{sC}]}{4\eta_R}\end{cases} \tag{C.15}$$

After solving Eq. (C.15), $a_1^{sC*}, b_1^{sC*}, a_2^{sC*}, b_2^{sC*}$ are derived as follows:

$$\begin{cases}a_1^{sC*}=\frac{\pi_M\theta}{\rho+\delta}, & b_1^{sC*}=\frac{[\gamma_M a_1^{sC*}]^2}{2\eta_M\rho}+\frac{\pi_M a}{\rho}+\frac{\gamma_R^2[2a_1^{sC*}+a_2^{sC*}]^2}{8\eta_R\rho}\\ a_2^{sC*}=\frac{\pi_R\theta}{\rho+\delta}, & b_2^{sC*}=\frac{\gamma_M^2 a_2^{sC*}a_1^{sC*}}{\eta_M\rho}+\frac{\pi_R a}{\rho}+\frac{\gamma_R^2 a_2^{sC*}[2a_1^{sC*}+a_2^{sC*}]}{4\eta_R\rho}\end{cases} \tag{C.16}$$

Substituting $a_1^{sC*}, b_1^{sC*}, a_2^{sC*}, b_2^{sC*}$ into Eq. (C.12), we can get the following function expressions:

$$V_M^{sC*}(G)=a_1^{sC*}G+b_1^{sC*} \quad and \quad V_R^{sC*}(G)=a_2^{sC*}G+b_2^{sC*} \tag{C.17}$$

Substituting (C.17) and its derivatives, we can get the equilibrium solutions of the manufacturer and the dealership.

$$E_M^{sC*}=\frac{\gamma_M\pi_M\theta}{\eta_M(\rho+\delta)}, X^{sC*}=\frac{2\pi_M-\pi_R}{2\pi_M+\pi_R}, E_R^{sC*}=\frac{\gamma_R(2\pi_M\theta+\pi_R\theta)}{2\eta_R(\rho+\delta)} \tag{C.18}$$

According to boundary conditions, substituting (C.18) into Eq. (2), the optimal trajectory of low-carbon reputation can be obtained. In addition, substituting (C.18) into Eqs. (22) and (23), the optimal profits manufacturer and dealership can be obtained. ■

Appendix D. Proof of Lemma 4

The proof process is the similar to Lemma 1 and the readers can easily derive it, so we omit it.

Appendix E. Proof of Proposition 5

From Lemma 4, we obtained the expression of E_R^{sD*}, which contain X^{sD*}. After replacing X^{sD*}, the expression of E_R^{sD*} can be written as

$$E_R^{sD*}=\frac{\gamma_R\pi_R\theta}{\eta_R\left(1-\frac{\pi_M}{\pi_M+\pi_R}\right)(\rho+\delta)}. \tag{E.1}$$

After simplifying the denominator of the right hand side of Eq. (E.1), finally, we can obtain Eq. (E.2).

$$E_R^{sD*}=\frac{\gamma_R(\pi_M+\pi_R)\theta}{\eta_R(\rho+\delta)}=E_R^{sA*}. \tag{E.2}$$

From Lemmas 2 and 3, we can obtain the expression of E_R^{sC*} and E_R^{sA*}, respectively. Then, we can obtain Eq. (E.3).

$$E_R^{sB*}-E_R^{sC*}=\frac{\gamma_R(2\pi_M\theta+\pi_R\theta)}{2\eta_R(\rho+\delta)}-\frac{\gamma_R\pi_R\theta}{\eta_R(\delta+\rho)}=\frac{\gamma_R\theta(2\pi_M-\pi_R)}{2\eta_R(\rho+\delta)}. \tag{E.3}$$

From the above equation, we can conclude that if $2\pi_M > \pi_R$, then $E_R^{sC*} < E_R^{sB*}$, otherwise, then $E_R^{sC*} \geq E_R^{sB*}$. ■

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